Properties of distinct-difference configurations and lightweight key predistribution schemes for grid-based networks

Simon R. Blackburn\textsuperscript{1} Keith M. Martin\textsuperscript{1} Tuvi Etzion\textsuperscript{2} Maura B. Paterson\textsuperscript{1}

\textsuperscript{1}Information Security Group
Royal Holloway, University of London

\textsuperscript{2}Technion -Israel Institute of Technology
Department of Computer Science

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Outline

Key Predistribution for Grid-Based Networks

Distinct-Difference Configurations
Precision Agriculture
Grid-Based Wireless Sensor Networks

- restricted memory
- restricted battery power
- restricted computational ability
- vulnerable to compromise
Key Predistribution

Definition (key predistribution scheme (KPS))
- nodes are assigned keys before deployment
- nodes that share keys can communicate securely

\[ \{k_1, k_5, k_7\} \quad \{k_3, k_5, k_{12}\} \]

E.g. Eschenauer and Gligor: Each node randomly draws \( m \) keys uniformly without replacement from a keypool \( \mathcal{K} \)
Goals for a KPS in a Grid-Based Network

- enable as many pairs of neighbouring nodes as possible to communicate securely
- minimise storage
- be resilient against node compromise

Observation: it is not necessary for two nodes to share more than one key
Costas Arrays

- one dot per row/column
- vector differences between dots are distinct
- applications to sonar, radar
- known constructions are based on finite fields
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Key Predistribution Using Costas Arrays

- uses an $n \times n$ Costas array
- each sensor stores $n$ keys
- each key is assigned to $n$ sensors
- two sensors share at most one key
- the distance between two sensors that share a key is at most $\sqrt{2}(n - 1)$
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Key Distribution for Grid-Based Networks

Distinct-Difference Configurations

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Distinct-Difference Configurations

**Definition (Distinct-Difference Configuration DD($m, r$))**

- $m$ dots are placed in a square grid
- the distance between any two dots is at most $r$
- vector differences between dots are all distinct

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\end{array}
\]

DD(5, 8)

- can be used for key predistribution in the same way as a Costas array
- more general than a Costas array $\Rightarrow$ more flexible choice of parameters
Upper Bounds on $m$

**Theorem**

If a DD$(m, r)$ exists, then

$$m \leq \frac{\sqrt{\pi}}{2} r + \frac{3\pi^{1/3}}{2^{5/3}} r^{2/3} + O(r^{1/3}) \approx 0.88623r + O(r^{2/3}).$$

- a DD$(m, r)$ is contained in an anticode $\mathcal{A}$ of diameter at most $r$ and area at most $(\pi/4)r^2$
- cover $\mathcal{A}$ in circles $\mathcal{C}$ of radius $\ell$
- count pairs $(\mathcal{C}, d)$ where $d$ is a pair of dots in $\mathcal{C} \cap \text{DD}(m, r)$
Lower Bounds on $m$

**Theorem**

There exists a $\text{DD}(m, r)$ with

$$m \approx 0.80795r - o(r).$$
Sequences with Distinct Differences

Definition

Let $A$ be an abelian group. A sequence $\{a_1, a_2, \ldots, a_m\} \subseteq A$ is a $B_2$-sequence if all the sums $a_{i_1} + a_{i_2}$ with $1 \leq i_1 \leq i_2 \leq m$ are distinct.

examples:

- Singer difference set
- Golomb ruler
- Bose: $B_2$-sequence of size $q$ in $\mathbb{Z}_{q^2-1}$
Folding a $B_2$-Sequence

\[ \{3, 13, 24, 29, 37, 41, 43, 44\} \]  
(mod 63)

\[
\begin{array}{cccccccc}
56 & 57 & 58 & 59 & 60 & 61 & 62 \\
48 & 49 & 50 & 51 & 52 & 53 & 54 & 55 \\
40 & 41 & 42 & 43 & 44 & 45 & 46 & 47 \\
32 & 33 & 34 & 35 & 36 & 37 & 38 & 39 \\
24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 \\
16 & 17 & 18 & 19 & 20 & 21 & 22 & 23 \\
8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\end{array}
\]
Results for the Manhattan Metric

Theorem

- If a $\overline{\text{DD}}(m, r)$ exists then $m \leq \frac{1}{\sqrt{2}} r + (3/2^{4/3}) r^{2/3} + O(r^{1/3})$.

- There exists a $\overline{\text{DD}}(m, r)$ with $m = \frac{1}{\sqrt{2}} r - o(r)$. 

![Grid Diagram](image-url)


http://www.isg.rhul.ac.uk/~uqah106/
thank you!