

Classifying and quantifying concept analyses

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ICFCA
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Outline

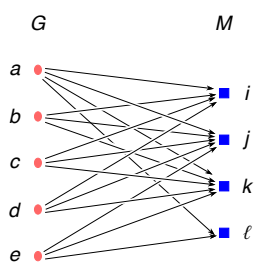
Two dimensions of concept analysis

views	<i>particle</i>	<i>wave</i>
<i>qualitative</i>	FCA	FSA
<i>quantitative</i>	QCA	LSA

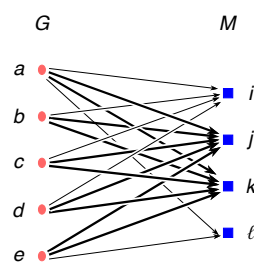
Two dimensions of concept analysis

views	<i>particle</i>	<i>wave</i>
<i>qualitative</i>	FCA	FSA
<i>quantitative</i>	QCA	LSA

A small town deep in the World Wild West

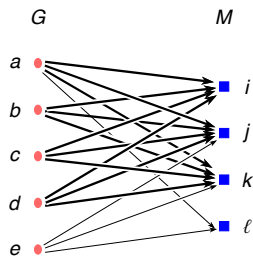


Concepts are complete subgraphs



Concepts are complete subgraphs

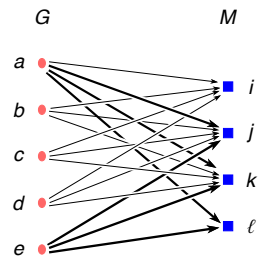
Classifying and
quantifying CA
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Navigation icons: back, forward, search, etc.

Concepts are complete subgraphs

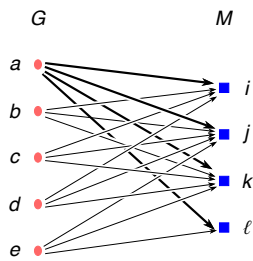
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Concepts are complete subgraphs

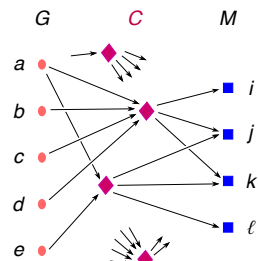
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Small town conceptualized

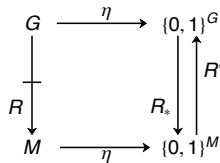
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Navigation icons: back, forward, search, etc.

Formal Concept Analysis

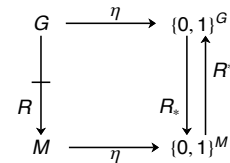
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Formal Concept Analysis

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$$R_* X = \bigcap_{x \in X} xR \quad \text{where } xR = \{y \in M \mid xRy\}$$

$$R^* Y = \bigcap_{y \in Y} Ry \quad \text{where } Ry = \{x \in G \mid xRy\}$$

Navigation icons: back, forward, search, etc.

Formal Concept Analysis

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Concepts

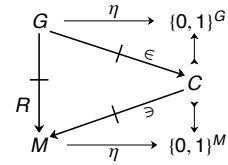
$$C = \{ \langle c_G, c_M \rangle \in \{0, 1\}^G \times \{0, 1\}^M \mid c_G = R^* c_M \wedge R_* c_G = c_M \}$$

Navigation icons

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Decomposition

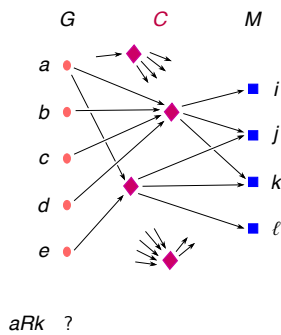


$$xRy = \bigvee_{c \in C} x \in c_G \wedge c_M \ni y$$

Navigation icons

Concept links "explain" context links

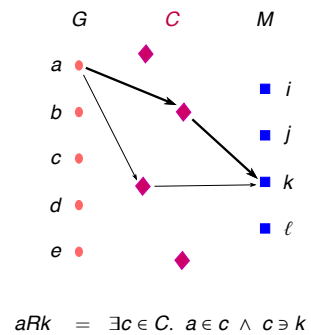
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Navigation icons

Concept links "explain" context links

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Navigation icons

Two dimensions of concept analysis

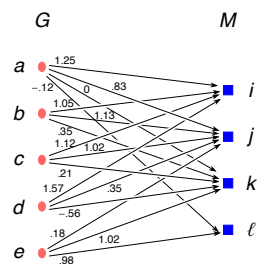
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views	particle	wave
qualitative	FCA	FSA
quantitative	QCA	LSA

Navigation icons

A small town deep in the World Wild West

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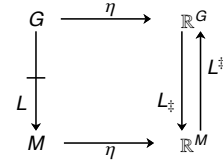
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	a	b	c	d	e
i	1.25	1.05	1.12	1.57	
j	.83	1.13	1.02	.35	.18
k	0	.35	.21	-.56	1.02
ℓ	-.12				.98

Navigation icons

Latent Semantic Analysis

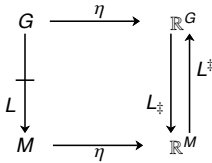
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$$L_{\ddagger} \xi = \left(\sum_{j=1}^n L_{ij} \xi_j \right)_{i=1}^m$$

$$L_{\ddagger} v = \left(\sum_{j=1}^m v_j L_{ij} \right)_{i=1}^n$$

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Concepts

$$C = \{ \langle \gamma_G, \gamma_M \rangle \in \overline{\mathbb{R}^G} \times \overline{\mathbb{R}^M} \mid \gamma_G = L_{\ddagger} \gamma_M \wedge L_{\ddagger} \gamma_G = \gamma_M \}$$

where $\overline{\mathbb{R}^X}$ is the set of rays in \mathbb{R}^X , i.e.

$$\overline{\mathbb{R}^X} = \mathbb{R}^X / \sim \quad \text{for } u \sim v \iff \exists r \neq 0. u = rv$$

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Concepts are the eigenspaces of $L_{\ddagger} L_{\ddagger}^{\ddagger}$ and $L_{\ddagger}^{\ddagger} L_{\ddagger}$

because

$$\gamma_G = L_{\ddagger} \gamma_M \wedge L_{\ddagger} \gamma_G = \gamma_M$$

$$\iff$$

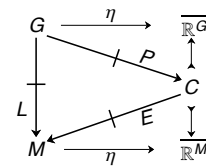
$$\gamma_G = L_{\ddagger} L_{\ddagger}^{\ddagger} \gamma_G \wedge L_{\ddagger}^{\ddagger} L_{\ddagger} \gamma_M = \gamma_M$$

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(Singular Value) Decomposition

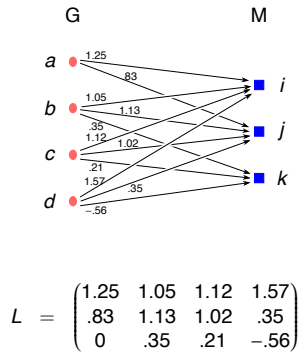


$$L_{ij} = \sum_{\gamma \in C} \lambda_{\gamma} \cdot E_{i\gamma} \cdot P_{\gamma j}$$

Navigation icons

Latent Semantics of the small town

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Navigation icons

Latent Semantics of the small town

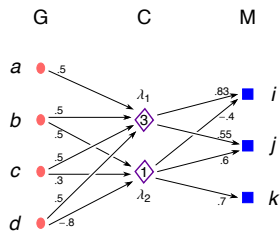
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$$\begin{pmatrix} 1.25 & 1.05 & 1.12 & 1.57 \\ .83 & 1.13 & 1.02 & .35 \\ 0 & .35 & .21 & -.56 \end{pmatrix} = \begin{pmatrix} .83 & -.4 \\ .55 & .6 \\ 0 & .7 \end{pmatrix} \cdot \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} .5 & 0 \\ .5 & .5 \\ .5 & .3 \\ .5 & -.8 \end{pmatrix}$$

Navigation icons

Latent Semantics of the small town

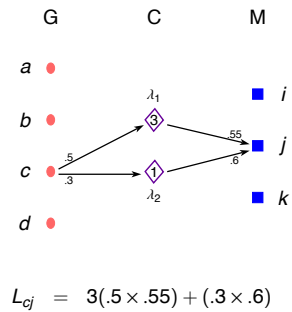
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Conceptual connections add up

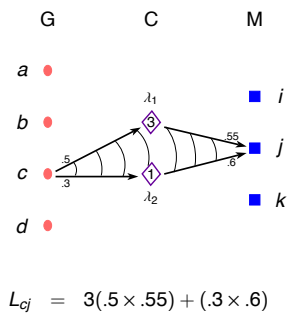
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Navigation icons

... and create waves of meaning

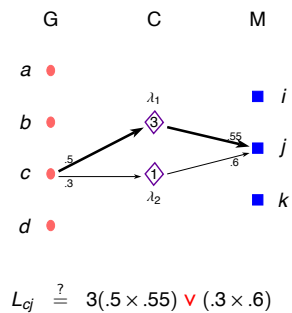
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Navigation icons

But what if we are need a single link?

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Navigation icons

Two dimensions of concept analysis

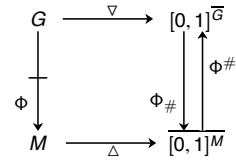
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views	particle	wave
qualitative	FCA	FSA
quantitative	QCA	LSA

Navigation icons

Quantitative Concept Analysis

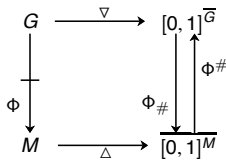
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Navigation icons

Quantitative Concept Analysis

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$$\Phi_{\#} \overleftarrow{\xi} = \left(\bigwedge_{j=1}^n \xi_j \rightarrow \phi_{ij} \right)_{i=1}^m$$

$$\Phi_{\#} \overrightarrow{v} = \left(\bigwedge_{i=1}^m v_i \rightarrow \phi_{ij} \right)_{j=1}^n$$

Navigation icons

Quantitative Concept Analysis

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Concepts

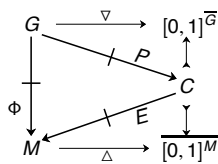
$$C = \{ \langle \overleftarrow{\gamma}, \overrightarrow{\gamma} \rangle \in [0, 1]^{\overline{G}} \times \overline{[0, 1]^M} \mid \overleftarrow{\gamma} = \Phi_{\#} \overrightarrow{\gamma} \wedge \Phi_{\#} \overleftarrow{\gamma} = \overrightarrow{\gamma} \}$$

Navigation icons

Quantitative Concept Analysis

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Decomposition



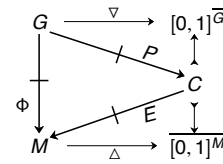
$$\Phi_{ij} = \bigvee_{\gamma \in C} E_{i\gamma} \cdot P_{\gamma j}$$

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Decomposition

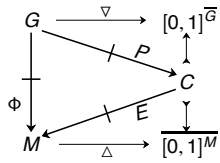


$$\left(i \vDash_{\Phi} j \right) = \bigvee_{\gamma \in C} \left(i \vDash_E \gamma \right) \cdot \left(\gamma \vDash_P j \right)$$

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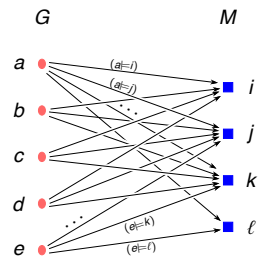
Quantitative Concept Analysis

Decomposition

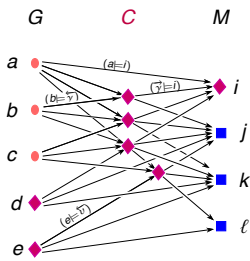


$$d_{\Phi}(i, j) = \bigwedge_{\gamma \in C} d_E(i, \gamma) + d_P(\gamma, j)$$

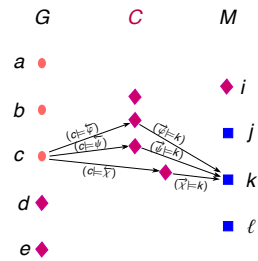
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A basis of quantified concepts

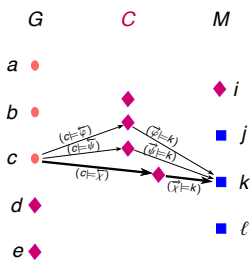


Strongest conceptual link \Leftarrow shortest path



$$(c=k) = (c=i) \cdot (i=k) \wedge (c=j) \cdot (j=k) \wedge (c=k) \cdot (k=k) \wedge (c=l) \cdot (l=k)$$

Strongest conceptual link \Leftarrow particle view



$$(c=k) = (c=i) \cdot (i=k) \wedge (c=j) \cdot (j=k) \wedge (c=k) \cdot (k=k) \wedge (c=l) \cdot (l=k)$$

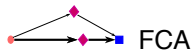
$$= (c=k) \cdot (k=k)$$

Overview

views	particle	wave
qualitative	FCA	FSA
quantitative	QCA	LSA

Overview

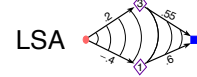
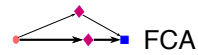
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Overview

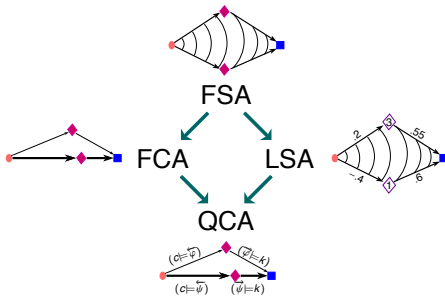
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Overview

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Overview

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Wave mechanics

- ▶ widest road
- ▶ induction
- ▶ evidence
- ▶ recommendation
- ▶ reputation
- ▶ ...

Particle mechanics

- ▶ shortest way
- ▶ deduction
- ▶ proof
- ▶ endorsement
- ▶ trust (PGP)
- ▶ ...

Navigation icons

Overview

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Wave mechanics

- ▶ widest road
- ▶ induction
- ▶ evidence
- ▶ recommendation
- ▶ reputation
- ▶ ...

Particle mechanics

- ▶ shortest way
- ▶ deduction
- ▶ proof
- ▶ endorsement
- ▶ trust (PGP)
- ▶ ...

FSA, LSA

FCA, QCA

Navigation icons

Future work

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- ▶ basis concepts
- ▶ algorithmics
- ▶ bridging the wave-particle duality in networks

Navigation icons

Future work

- ▶ basis concepts
- ▶ algorithmics
- ▶ bridging the wave-particle duality in networks
- ▶ everything else

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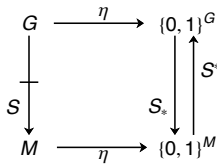
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Appendix

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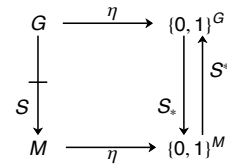
Formal Semantic Analysis



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Formal Semantic Analysis



$$S_* X = \bigcup_{x \in X} xS \quad \text{where } xS = \{y \in M \mid xSy\}$$

$$S^* Y = \bigcap_{y \in \bar{Y}} \bar{S}y \quad \text{where } \bar{S}y = \{x \in G \mid \neg xSy\} \text{ and } \bar{Y} = B \setminus Y$$

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Formal Semantic Analysis

Concepts

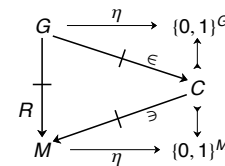
$$C = \{ \langle c_G, c_M \rangle \in \{0,1\}^G \times \{0,1\}^M \mid c_G = S^* c_M \wedge S_* c_G = c_M \}$$

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Formal Concept Analysis

Decomposition

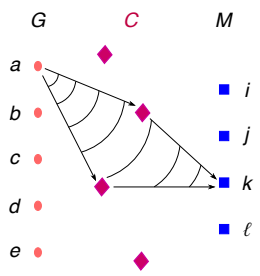


$$xRy = \bigwedge_{c \in C} x \in c_G \Rightarrow c_M \ni y$$

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Leibniz-style conceptualization



$$aSk = \forall c \in C. a \in c \Rightarrow c \ni k$$