Coalgebra of market games

Dusko Pavlovic

Kestrel Institute
and
Oxford University

CALCO, Udine
September 2009

Outline

Outline

A process is a coalgebra

$$X \xrightarrow{R} A \Rightarrow M(B \times X)$$

where

- A — controls
- B — measurements
- X — states

A process is a coalgebra

$$A \times X \xrightarrow{R} M(B \times X)$$

where

- A — controls
- B — measurements
- X — states

A process is a coalgebra

$$A \times X \xrightarrow{R} B \times X$$

where

- A — controls
- B — measurements
- X — states

Feedback and stable control

$$\frac{\dfrac{A\times X \overset{R}{\nrightarrow} B\times X \qquad\qquad B\times X \overset{\phi}{\nrightarrow} A}{A\times X \overset{R}{\nrightarrow} B\times X \overset{\phi}{\nrightarrow} A}}{\begin{array}{ccc} X & \xrightarrow{\ \gamma=Fix_A(\phi\circ R)\ } & A \\ {\scriptstyle\langle\gamma,\mathrm{id}\rangle}\downarrow & & \uparrow{\scriptstyle\phi} \\ A\times X & \xrightarrow[R]{} & B\times X \end{array}}$$

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Conclusion

A game is a coalgebra

$$A\times X \overset{\varrho}{\nrightarrow} B\times X$$

where

- $A = \prod_{i\in n} A_i$ — moves
- $B = \prod_{i\in n} B_i$ — values
- $X = \prod_{i\in n} X_i$ — positions
- $n = \{0, 1, \ldots, n-1\}$ — players

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Conclusion

A game is a coalgebra

$$A\times X \overset{\varrho}{\nrightarrow} B\times X$$

where

- $A = \prod_{i\in n} A_i$ — moves
- $B = \prod_{i\in n} B_i$ — values (ordered ring)
- $X = \prod_{i\in n} X_i$ — positions
- $n = \{0, 1, \ldots, n-1\}$ — players

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Conclusion

Response strategy and equilibrium

$$\frac{\dfrac{\dfrac{A\times X \overset{\varrho}{\nrightarrow} B\times X}{A_{-i}\times X_i \xrightarrow{\ RS_i\ } A_i}}{A\times X \xrightarrow{\ RS=\langle RS_i\circ\pi_i\rangle_{i\in m}\ } A}}{\begin{array}{ccc} X & \overset{RS^\bullet=Fix_A(RS)}{\nrightarrow} & A \\ & {\scriptstyle\langle RS^\bullet,\mathrm{id}\rangle}\searrow \quad \nearrow{\scriptstyle RS} & \\ & A\times X & \end{array}}$$

where

$$A_{-i} \;=\; \prod_{\substack{k\in n\\ k\neq i}} A_k$$

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Conclusion

Tasks for coalgebra of games

- analyze positions as states in X
  - coalgebra homomorphisms between games
  - position bisimilarity

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Conclusion

Tasks for coalgebra of games

- analyze positions as states in X
  - coalgebra homomorphisms between games
  - position bisimilarity
- construct equilibria as fixed points in A
  - static $1 \overset{RS^\bullet}{\nrightarrow} A$ or position-wise $X \overset{RS^\bullet}{\nrightarrow} A$
  - equilibrium at a stationary position $1 \overset{\langle RS^\bullet, x\rangle}{\nrightarrow} A\times X$

Tasks for coalgebra of games

- analyze positions as states in X
  - coalgebra homomorphisms between games
  - position bisimilarity
- construct equilibria as fixed points in A
  - static $1 \xrightarrow{RS^\bullet} A$ or position-wise $X \xrightarrow{RS^\bullet} A$
  - equilibrium at a stationary position $1 \xrightarrow{\langle RS^\bullet, x\rangle} A \times X$

Outline

Standard notions
(Static: $X = 1$)

A1 - Best response strategy $RS = BR$

$$s_{-i}\ BR_i\ s_i \iff \forall t_i \in A_i.\ \varrho_i(t_i, s_{-i}) \leq \varrho_i(s_i, s_{-i})$$

Standard notions
(Static: $X = 1$)

A1 - Best response relation

$$s\ BR\ t \iff \forall i \in n.\ s_{-i}\ BR_i\ t_i$$

Standard notions
(Static: $X = 1$)

A2 - Nash equilibrium

$$BR^\bullet s \iff s\ BR\ s$$
$$\iff \forall i \in n.\ s_{-i}\ BR_i\ s_i$$

Standard notions
(Static: $X = 1$)

A3 - Rationalizable (undominated) profile

$$BR^* s \iff \exists t.\ BR^* t \wedge tBRs$$

Standard notions

Upshot

Nash equilibrium is

- a joint result of individual optimizations
- a social solution of a distributed problem
- noone can improve their gain on their own
- it leads beyond the "zero sum" view of the world

Standard notions

Issues

- existence of equilibrium
  - Q: Is $BR^\bullet$ empty?
- equilibrium selection
  - Q: What if $s, t \in BR^\bullet$, and i plays s_i and j plays t_j?
- social benefit of equilibrium
  - Q: Is $\sum_i \varrho_B^i(s)$ a global maximum (Pareto optimal)?

Standard notions

Issues

- existence of equilibrium
  - A: Kakutani theorem
- equilibrium selection
  - A: attractor dynamics
- social benefit of equilibrium
  - A: **program the notions of response**

Example: Bird Politics

(Chicken, Missile Crisis, Prisoners' Dilemma...)

For

- $i \in 2 = \{0, 1\}$
- $A_i = \{retreat, attack\}$
- $B_i = \mathbb{R}$

the payoff $A_0 \times A_1 \xrightarrow{\varrho} B_0 \times B_1$ is given by

	retreat	attack
retreat	$\frac{w}{2}$; $\frac{w}{2}$	0 ; w
attack	w ; 0	$\frac{w}{2} - c$; $\frac{w}{2} - c$

Example: Bird Politics

(Chicken, Missile Crisis, Prisoners' Dilemma...)

If $c < \frac{w}{2}$, the only Nash equilibrium $1 \xrightarrow{s} A_0 \times A_1$ is

$$s = \langle \text{attack}, \text{attack} \rangle$$

with the social gain of

$$\sum \varrho(s) = w - 2c$$

Example: Bird Politics

(Chicken, Missile Crisis, Prisoners' Dilemma...)

The *unstable* profile

$$r = \langle \text{retreat}, \text{retreat} \rangle$$

would give the social gain of

$$\sum \varrho(s) = w$$

Static strategies
($X = 1$)

B1 - Stable strategy $RS = SR$

$$s_{-i}\ SR_i\ s_i \iff \forall t \in A\ \forall \varepsilon > 0.\ (1-\varepsilon)\varrho_i(t_i, s_{-i}) + \varepsilon\varrho_i(t_i, t_{-i}) \leq (1-\varepsilon)\varrho_i(s_i, s_{-i}) + \varepsilon\varrho_i(s_i, t_{-i})$$

Static strategies
($X = 1$)

B1 - Stable strategy $RS = SR$

$$s_{-i}\ SR_i\ s_i \iff \forall t_i \in A_i.\ \varrho_i(t_i, s_{-i}) \leq \varrho_i(s_i, s_{-i}) \wedge (\varrho_i(t_i, s_{-i}) = \varrho_i(s_i, s_{-i}) \Rightarrow \forall t_{-i} \in A_{-i}.\ \varrho_i(t_i, t_{-i}) \leq \varrho_i(s_i, t_{-i}))$$

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Conclusion

Static strategies
($X = 1$)

B1 - Stable response relation

$$s\ SR\ t \iff \forall i \in n.\ s_{-i}\ SR_i\ t_i$$

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Conclusion

Static strategies
($X = 1$)

B2 - Stable equilibrium

$$\begin{aligned} SR^{\bullet}s &\iff \forall i \in n.\ s_{-i}\ SR_i\ s_i \\ &\iff \forall t \in A. \forall i \in n.\ \varrho_i(t_i, s_{-i}) \leq \varrho_i(s_i, s_{-i}) \wedge \\ &\quad \varrho_i(s_i, t_{-i}) = \varrho_i(s_i, s_{-i}) \Rightarrow \\ &\quad \forall t_{-i} \in A_{-i}.\ \varrho_i(t_i, t_{-i}) \leq \varrho_i(s_i, t_{-i}) \end{aligned}$$

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Conclusion

Static strategies
($X = 1$)

B3 - Stable (admissible) profile

$$SR^{*}s \iff \exists t.SR^{*}t \wedge tSRs$$

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Conclusion

Example: Bird Politics
(Chicken, Missile Crisis, Prisoners' Dilemma. . .)

If $c < \frac{w}{2}$, the only stable profile $1 \xrightarrow{s} A_0 \times A_1$ is

$$s = \langle \text{attack}, \text{attack} \rangle$$

with the social gain of

$$\sum \varrho(s) = w - 2c$$

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Conclusion

Static strategies
($X = 1$)

C1 - Uniform strategy $RS = UR$

$$s_{-i}\ UR_i\ s_i \iff s_{-i}\ BR_i\ s_i \wedge \forall t_{-i} \in A_{-i}.\ s_i\ BR_{-i}\ t_{-i} \Rightarrow t_{-i}\ BR_i\ s_i$$

Static strategies
($X = 1$)

C2 - Uniform equilibrium

$$UR^{\bullet}\ s \iff \forall i \in n.\ s_{-i}\ UR_i\ s_i \iff BR^{\bullet}\ s \wedge \forall i \in n.\ \forall t_{-i} \in A_{-i}.\ s_i\ BR_{-i}\ t_{-i} \Rightarrow t_{-i}\ BR_i\ s_i$$

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Conclusion

Static strategies
($X = 1$)

C3 - Uniform profile

$$UR^{*}\ s \iff \exists t.UR^{*} t \wedge tURs$$

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Conclusion

Example: Bird Politics
(Chicken, Missile Crisis, Prisoners' Dilemma...)

If $c < \frac{w}{2}$, there is no uniform equilibrium for Bird Politics (neither pure nor mixed).

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Conclusion

Example: Bird Politics
(Chicken, Missile Crisis, Prisoners' Dilemma...)

If $c < \frac{w}{2}$, there is no uniform equilibrium for Bird Politics (neither pure nor mixed).

The Kakutani theorem does not apply because the uniform response relation is not convex.

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Conclusion

Static strategies
($X = 1$)

D1 - Constructive strategy $RS = CR$

$$s_{-i}\ CR_i\ s_i \iff \forall t_i \in A_i.\ \varrho_i(s_i, s_{-i}) < \varrho_i(t_i, s_{-i}) \Rightarrow \exists t_{-i} \in A_{-i}.\ \varrho_{-i}(t_i, t_{-i}) > \varrho_{-i}(t_i, s_{-i}) \wedge \varrho_i(t_i, t_{-i}) < \varrho_i(s_i, s_{-i})$$

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Conclusion

Static strategies
($X = 1$)

D2 - Constructive equilibrium

$$CR^\bullet\ s \quad \Longleftrightarrow \quad \forall i. s_{-i} CR_i s_i$$

Static strategies
($X = 1$)

D3 - Uniform profile

$$CR^*\ s \quad \Longleftrightarrow \quad \exists t. CR^* t \wedge tCRs$$

Example: Bird Politics
(Chicken, Missile Crisis, Prisoners' Dilemma. . .)

There is a constructive equilibrium, consisting of the mixed strategies favoring the Pareto optimal solution $\langle retreat, retreat\rangle$.

Outline

Coalgebra of games

Equilibrium programming

Position analysis

- Problem of coordination
- Problem of competition

Conclusion

Tasks for coalgebra of games

- analyze positions as states in X
  - coalgebra homomorphisms between games
  - position bisimilarity
- construct equilibria as fixed points in A
  - static $1 \xrightarrow{RS^\bullet} A$ or position-wise $X \xrightarrow{RS^\bullet} A$
  - equilibrium at a stationary position $1 \xrightarrow{\langle RS^\bullet, x\rangle} A \times X$

Types-as-positions: Bird Politics
(Chicken, Missile Crisis, Prisoners' Dilemma. . .)

For

- $i \in 2 = \{0, 1\}$
- $A_i = \{retreat, attack\}$
- $B_i = \mathbb{R}$
- $X_i = [0, 1]$
  - $x_0 = \mathrm{Prob}(a_1 = retreat)$
  - $x_1 = \mathrm{Prob}(a_0 = retreat)$

the payoff $A_i \times X_i \xrightarrow{\varrho^i_B} B_i$ is given by

$$\varrho^i_B(\text{retreat}, x_i) = x_i \frac{w}{2}$$

$$\varrho^i_B(\text{attack}, x_i) = x_i \frac{w + 2c}{2} + \frac{w - 2c}{2}$$

Types-as-positions: Bird Politics

(Chicken, Missile Crisis, Prisoners' Dilemma. . .)

[. . .]

Example: Majority game

(static: $X = 1$)

For

- $i \in 2m+1 = \{0, 1, \ldots 2m\}$ — players
- $A_i = \{\blacktriangleleft, \blacktriangleright\}$ — moves
- $B_i = \{0, 1\}$ — values

the payoff $A \xrightarrow{\varrho} B$ is

$$\varrho^i(s) = \begin{cases} 1 & \text{if } \#\{j \mid s_j = s_i\} > m \\ 0 & \text{otherwise} \end{cases}$$

Example: Majority game

(static: $X = 1$)

The equilibrium $1 \xrightarrow{s} A$ consists of the mixed strategies

$$s_i = \frac{1}{2}\blacktriangleleft + \frac{1}{2}\blacktriangleright$$

with the expected social gain of

$$\mathbb{E}\Big(\sum \varrho(s)\Big) = 2\sum_{k=m+1}^{2m+1} \frac{\binom{2m+1}{k}}{2^{2m+1}} k = m + \frac{1}{2}$$

Positions for coordination: Majority game

(with 1-step memory)

For

- $i \in 2m+1 = \{0, 1, \ldots 2m\}$ — players
- $A_i = \{\blacktriangleleft, \blacktriangleright\}$ — moves
- $B_i = \{0, 1\}$ — values
- $X_i = \{\blacktriangleleft, \blacktriangleright\}$ — positions
  - $\xi \in X = \prod X_i$ — initialized randomly

the game $A \times X \xrightarrow{\varrho} B \times X$ becomes

$$\varrho^i_B(s, x) = \begin{cases} 1 & \text{if } \#\{j \mid s_j = s_i\} > m \\ 0 & \text{otherwise} \end{cases}$$

$$\varrho^i_X(s, x) = \diamond \quad \text{such that } \#\{j \mid s_j = \diamond\} > m$$

Positions for coordination: Majority game

The equilibria $X \xrightarrow{s} A$ are the coordination policies

$$s_i(x) = x$$
$$s'_i(x) = \neg x$$

which assure the social gain of

$$\sum \varrho(s) = 2m+1$$

Example: Minority game

(static: $X = 1$)

For

- $i \in 2m+1 = \{0, 1, \ldots 2m\}$ — players
- $A_i = \{\blacktriangleleft, \blacktriangleright\}$ — moves
- $B_i = \{0, 1\}$ — values

the payoff $A \xrightarrow{\varrho} B$ is

$$\varrho^i(s) = \begin{cases} 1 & \text{if } \#\{j \mid s_j = s_i\} \leq m \\ 0 & \text{otherwise} \end{cases}$$

Example: Minority game

(static: $X = 1$)

The static equilibrium $1 \overset{s}{\longrightarrow} A$ is again

$$s_i \quad = \quad \frac{1}{2}\blacktriangleleft + \frac{1}{2}\blacktriangleright$$

with the expected social gain of

$$\mathbb{E}\left(\sum \varrho(s)\right) \quad = \quad \sum_{k=1}^{m} \frac{\binom{2m+1}{k}}{2^{2m+1}} k \quad = \quad \frac{m}{2}$$

Example: Minority game

(static: $X = 1$)

- Even with the maximal gain of $\sum \varrho(s) = m$, there are $m+1$ players with $\varrho^i(s) = 0$.

Example: Minority game

(static: $X = 1$)

- Even with the maximal gain of $\sum \varrho(s) = m$, there are $m+1$ players with $\varrho^i(s) = 0$.
- There is always a majority with an incentive to disturb the current state.

Example: Minority game

(static: $X = 1$)

- Even with the maximal gain of $\sum \varrho(s) = m$, there are $m+1$ players with $\varrho^i(s) = 0$.
- There is always a majority with an incentive to disturb the current state.
- This leads **from equilibrium to evolution**.

Positions for stabilization: Minority game

(with ℓ-step memory and d ideas)

Suppose that the player i sees the positions from

$$X_i \quad = \quad M \times S^{1+d} \times \ell$$

where

- $M = S = \{\blacktriangleleft, \blacktriangleright\}^{\ell}$ — memory, strategies, ideas
- $d = \{0, 1, \ldots, d-1\}$ — number of ideas
- $\ell = \{0, 1, \ldots, \ell - 1\}$ — length of the memory

Positions for stabilization: Minority game

(with ℓ-step memory and d ideas)

A position

$$x_i \;=\; \langle \mu, \sigma^{i0}, \sigma^{i1}, \ldots, \sigma^{id}, k \rangle \;\in\; M \times S^{1+d} \times \ell \;=\; X_i$$

records

- μ — the recent ℓ minority (winning) choices
- σ^{i0} — i's current strategy (ℓ-tuple of choices)
- $\sigma^{i1}, \sigma^{i2}, \ldots, \sigma^{id}$ — i's bag of ideas for strategies
- k — the current moment in the ℓ-cycle history

Positions for stabilization: Minority game

For

- $i \in 2m+1 = \{0, 1, \ldots 2m\}$ — players
- $A_i = \{◄, ►\}$ — moves
- $B_i = \{0, 1\}$ — values
- $X_i = M \times S^{1+d} \times \ell$ — positions
  - $\xi^i \in X_i$ — initialized randomly

the payoff $A \times X \xrightarrow{\varrho_B} B$ remains

$$\varrho_B^i(s, x) \quad = \quad \begin{cases} 1 & \text{if } \#\{j \mid s_j = s_i\} \ \leq \ m \\ 0 & \text{otherwise} \end{cases}$$

Positions for stabilization: Minority game

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Coordination
Competition
Conclusion

...while the position update $A \times X_i \xrightarrow{\varrho_X^i} X_i$ maps

$$\varrho_X^i(s, \ \langle \mu, \sigma^{i*}, k \rangle) \quad = \quad \langle \tilde{\mu}, \tilde{\sigma}^{i*}, \tilde{k} \rangle$$

so that

- $\tilde{k} = k + 1 \mod \ell$
- $\tilde{\mu} = \langle \diamond, \mu_0, \mu_1, \ldots, \mu_{\ell-2} \rangle$
  - where $\diamond$ is the minority choice, i.e. $\#\{j \mid s_j = \diamond\} \leq m$
- $\tilde{\sigma}^{i*}$ is obtained by reordering
  - $\hat{\sigma}^{i*} = \langle \sigma^{i*}_{\ell-1}, \sigma^{i*}_0, \sigma^{i*}_1, \ldots, \sigma^{i*}_{\ell-2} \rangle$
  - to maintain the invariant
    $$\Delta(\tilde{\sigma}^{i0}, \tilde{\mu}) \leq \Delta(\tilde{\sigma}^{i1}, \tilde{\mu}) \leq \cdots \leq \Delta(\tilde{\sigma}^{id}, \tilde{\mu})$$

Positions for stabilization: Minority game

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Coordination
Competition
Conclusion

...while the position update $A \times X_i \xrightarrow{\varrho_X^i} X_i$ maps

$$\varrho_X^i(s, \ \langle \mu, \sigma^{i*}, k \rangle) \quad = \quad \langle \tilde{\mu}, \tilde{\sigma}^{i*}, \tilde{k} \rangle$$

so that

- $\tilde{k} = k + 1 \mod \ell$
- $\tilde{\mu} = \langle \diamond, \mu_0, \mu_1, \ldots, \mu_{\ell-2} \rangle$
  - where $\diamond$ is the minority choice, i.e. $\#\{j \mid s_j = \diamond\} \leq m$
- $\tilde{\sigma}^{i*}$ is obtained by reordering
  - $\hat{\sigma}^{i*} = \langle \sigma^{i*}_{\ell-1}, \sigma^{i*}_0, \sigma^{i*}_1, \ldots, \sigma^{i*}_{\ell-2} \rangle$
  - to maintain the invariant
    $$\Delta(\tilde{\sigma}^{i0}, \tilde{\mu}) \leq \Delta(\tilde{\sigma}^{i1}, \tilde{\mu}) \leq \cdots \leq \Delta(\tilde{\sigma}^{id}, \tilde{\mu})$$

  — thus $\tilde{\sigma}^{i0}$ is the best and $\tilde{\sigma}^{id}$ the worst strategy w.r.t. $\tilde{\mu}$

Positions for stabilization: Minority game

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Coordination
Competition
Conclusion

Let the profile $X \xrightarrow{s} A$ be defined by

$$s_i(\mu, \sigma^{i*}, k) \quad = \quad \sigma^{i0}_k$$

i.e., each player plays his currently best strategy.

Positions for stabilization: Minority game

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Coordination
Competition
Conclusion

Evolution: refine $A \times X_i \xrightarrow{\varrho_X^i} X_i$

- Each player *randomly mutates* her state by
  - dropping her worst idea $\sigma^{i(\ell-1)} \in \{◄, ►\}^\ell$
  - adding a random idea $\sigma' \in \{◄, ►\}^\ell$.

  at chosen intervals, or triggered by bad scores.

Positions for stabilization: Minority game

Coalgebra of market games
D. Pavlovic
Games
Equilibria
Positions
Coordination
Competition
Conclusion

Evolution: refine $A \times X_i \xrightarrow{\varrho_X^i} X_i$

- Each player *randomly mutates* her state by
  - dropping her worst idea $\sigma^{i(\ell-1)} \in \{◄, ►\}^\ell$
  - adding a random idea $\sigma' \in \{◄, ►\}^\ell$.

  at chosen intervals, or triggered by bad scores.
- This leads to jointly stable populations of players.

Positions for stabilization: Minority game

Stable positions

$$\frac{\dfrac{X \xrightarrow{\langle s,\mathrm{id}\rangle} A\times X \qquad A\times X \xrightarrow{\varrho_X} X}{X \xrightarrow{\langle s,\mathrm{id}\rangle} A\times X \xrightarrow{\varrho_X} X}}{\begin{array}{ccc} 1 & \xrightarrow{\eta=Fix_X(\varrho_X\circ s)} & X \\ \eta\downarrow & & \uparrow\varrho_X \\ X & \xrightarrow[\langle s,\mathrm{id}\rangle]{} & A\times X \end{array}}$$

yield improved social gains $\mathbb{E}(\Sigma\varrho) > \frac{m}{2} + q$.

Example: Market game

(static: $X = 1$)

For

- $i \in n = \{0, 1, \ldots n-1\}$ — players (sellers, producers)
- $A_i = B_i = \mathbb{R}$ — moves, values

the payoff $A \xrightarrow{\varrho} B$ is

$$\varrho^i(s) = \begin{cases} s_i - c_i & \text{if } \forall j_{\in n\setminus\{i\}}.\ s_i < s_j \\ 0 & \text{otherwise} \end{cases}$$

Example: Market game

(static: $X = 1$)

For

- $i \in n = \{0, 1, \ldots n-1\}$ — players (sellers, producers)
- $A_i = B_i = \mathbb{R}$ — moves, values

the payoff $A \xrightarrow{\varrho} B$ is

$$\varrho^i(s) = \begin{cases} s_i - c_i & \text{if } \forall j_{\in n\setminus\{i\}}.\ s_i < s_j \\ 0 & \text{otherwise} \end{cases}$$

where

- s_i is the market price offered by the producer i,
- c_i is the production cost of i

Example: Market game

(static: $X = 1$)

The equilibria $1 \xrightarrow{s} A$ consist of the strategies

$$s_i = c_i + \varepsilon_i$$

where $\varepsilon_i \in [p_i, q_i]$ is the desired profit.

Example: Market game

(with memory and tactics)

Marketing tactics (equilibrium selection)

- to win, find ε such that $c_i + \varepsilon < c_j + \varepsilon_j$ for all $j \neq i$

Example: Market game

(with memory and tactics)

Marketing tactics (equilibrium selection)

- to win, find ε such that $c_i + \varepsilon < c_j + \varepsilon_j$ for all $j \neq i$
  - to profit, maximize among such ε

Example: Market game
(with memory and tactics)

Marketing tactics (equilibrium selection)

- to win, find ε such that $c_i + \varepsilon < c_j + \varepsilon_j$ for all $j \neq i$
  - to profit, maximize among such ε
- change the game:
  - sway the buyer to pay more than the lowest price
    - lock in, bundling, price discrimination. . .
  - manipulate the market information
    - advertising, branding. . .

Stable solution: Second price market game
(Static: $X = 1$)

For

- $i \in n = \{0, 1, \ldots n-1\}$ — players
- $A_i = B_i = \mathbb{R}$ — moves, values

the payoff $A \overset{\varrho}{\rightarrow} B$ is

$$\varrho^i(s) = \begin{cases} \lceil s_i \rceil^s - s_i & \text{if } \forall j_{\in n \setminus \{i\}}.\ s_i < s_j \\ 0 & \text{otherwise} \end{cases}$$

Stable solution: Second price market game
(Static: $X = 1$)

For

- $i \in n = \{0, 1, \ldots n-1\}$ — players
- $A_i = B_i = \mathbb{R}$ — moves, values

the payoff $A \overset{\varrho}{\rightarrow} B$ is

$$\varrho^i(s) = \begin{cases} \lceil s_i \rceil^s - s_i & \text{if } \forall j_{\in n \setminus \{i\}}.\ s_i < s_j \\ 0 & \text{otherwise} \end{cases}$$

where

$$\lceil a \rceil^\beta = \bigwedge \{b \in \beta \mid a < b\}$$

Stable solution: Second price market game
(Static: $X = 1$)

The unique equilibrium $1 \overset{s}{\rightarrow} A$ consists of the strategies

$$s_i = c_i$$

Stable solution: Second price market game
(Static, stable, unimplementable)

The unique equilibrium $1 \overset{s}{\rightarrow} A$ consists of the strategies

$$s_i = c_i$$

i.e.,

- each player announces her production cost
- the lowest cost wins the market
- the profit is $\lceil c_i \rceil^c - c_i$
  - the second lowest cost – the lowest cost

Outline

Conclusion: Tasks for coalgebra of games

- analyze positions as states in X
  - coalgebra homomorphisms between games
  - position bisimilarity
- construct equilibria as fixed points in A
  - static $1 \xrightarrow{s} A$ or position-wise $X \xrightarrow{s} A$
  - equilibrium at a stationary position $1 \xrightarrow{\langle s,x \rangle} A \times X$