Bayesian authentication with imperfect cryptography in pervasive networks

Dusko Pavlovic
(in ongoing collaboration with Cathy Meadows)

June 2009

Outline

Problem of proximity authentication

Flavours of authentication

Proving proximity authentication

Conclusions and ongoing work

EMV payment system

Figure: Chip & Pin authentication

Problem

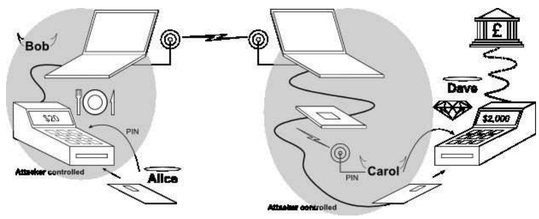


Figure: Smart card relay attack [Anderson et al.]

Problem

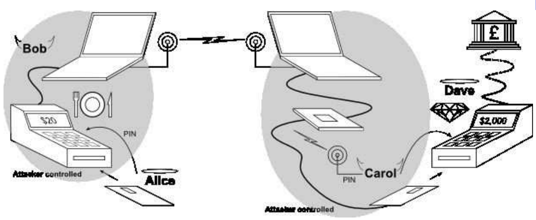


Figure: Smart card relay attack [Anderson et al.]

This will become much easier with NFC phones!

Problem

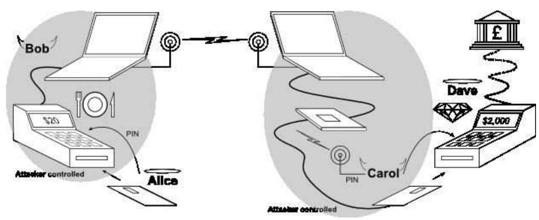


Figure: Smart card relay attack [Anderson et al.]

This will become much easier with NFC phones!

But the NFC phones have a timer!

Task

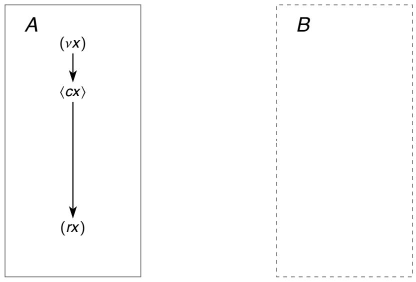
Strengthen authentication

Verify for each principal

- not only **certificates**
- but also **proximity**

What is authentication?

Challenge-Response pattern



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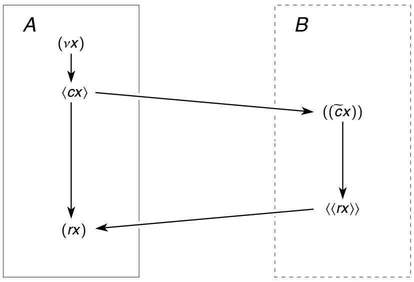
Challenge-Response pattern



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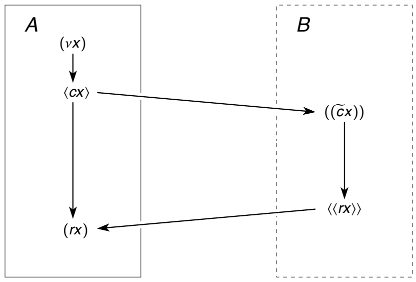
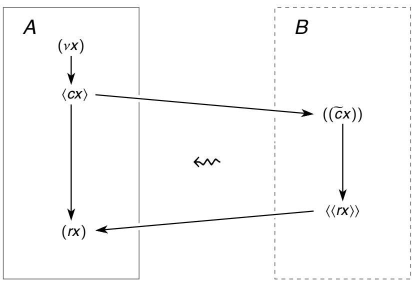


$\text{Local}_A \implies \text{Global}_{AB}$

What is authentication?

Challenge-Response pattern



$\text{Local}_A \implies \text{Global}_{AB}$

Derive global from local?!

Derive global from local?!

Problem

"There is no logical impossibility in the hypothesis that the world sprang into being five minutes ago, exactly as it then was, with a population that 'remembered' a wholly unreal past."

Bertrand Russell, The Analysis of Mind

Derive global from local?!

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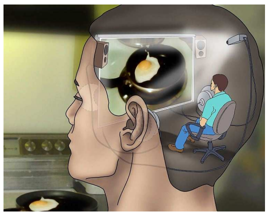
Philosophical solution: reflection



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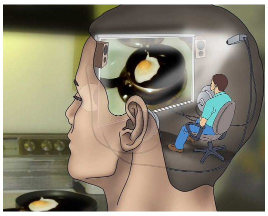

René to himself: "I think, therefore I exist."

Derive global from local?!

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Computational solution: cheating is hard

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Computational solution: cheating is hard

Alan to Machine: "You are a machine."

Derive global from local?!

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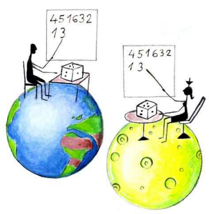
Cryptographic solution: authenticity from secrecy



Alice to Bob: "Nobody else could decrypt this, therefore you exist."

Authentication with perfect cryptography

"Theorem"

Suppose that only Bob knows k^B, such that

- rx can be computed from $\widetilde{c}x$ and k^B
- this is the only way to compute rx here

Authentication with perfect cryptography

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Then $\text{Local}_A \Longrightarrow \text{Global}_{AB}$ holds.

Authentication with perfect cryptography

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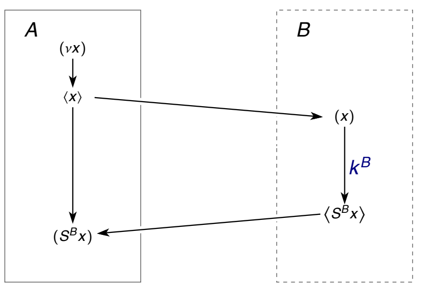
Example



$\text{Local}_A \qquad \Longrightarrow \qquad \text{Global}_{AB}$

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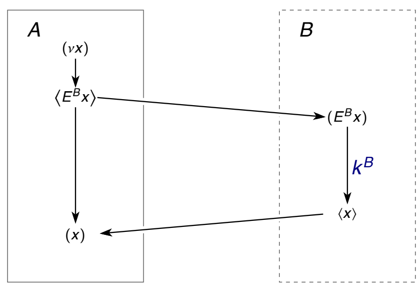

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Authentication in Protocol Logics

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Theorem A

Suppose that only Bob knows k^B, such that

- rx can be computed from $\widetilde{c}x$ and k^B
  - $k^B, \widetilde{c}x \vdash rx$
- this is the only way to compute rx here
  - $\{\!\{k^B\}\!\}$ guards rx within CR

Then $\text{Local}_A \Longrightarrow \text{Global}_{AB}$ holds where

$$\begin{aligned} \text{Local}_A &= (\nu x)_A \to \langle cx\rangle_A \to (rx)_A \\ \text{Global}_{AB} &= (\nu x)_A \to \langle cx\rangle_A \to ((\widetilde{c}x))_B \to \langle\langle rx\rangle\rangle_B \to (rx)_A \end{aligned}$$

Authentication in Protocol Logics

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Algebraic guards

Definition

For $s \in \mathbb{T}[V]$ and $\mathcal{G}, \mathcal{C} \in \wp\wp\mathbb{T}[V]$ with $s \not\sqsubseteq \Theta \in \mathcal{C}$ define

$$\mathcal{G} \text{ guards } s \text{ within } \mathcal{C}$$

$$\Updownarrow$$

$$\forall \Theta \in \mathcal{C}\ \exists \Gamma \in \mathcal{G}.\quad \Theta \vdash s \Longrightarrow \Theta \vdash \Gamma$$

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where

$$\begin{aligned} \Theta \vdash \Gamma &\iff \forall s \in \Gamma \exists \vec{t} \subseteq \Theta \exists f(\vec{x}) \in \mathcal{F}.\ f(\vec{t}) = s \\ \Theta \sqsupseteq \Gamma &\iff \forall s \in \Gamma \exists \vec{t} \subseteq \Theta \exists j(\vec{x}) \in \mathcal{J}.\ j(\vec{t}) = s \end{aligned}$$

Algebraic guards

Examples

- $\{\{x, g^y\}, \{y, g^x\}\}$ guards g^{xy} within DH

Algebraic guards

Examples

- $\{\{x, g^y\}, \{y, g^x\}\}$ guards g^{xy} within DH
- $\left\{\{\overline{k}\}, \{m\}\right\}$ guards m within $\{A \text{ to } B : E(k, m)\}$

Perfect cryptography

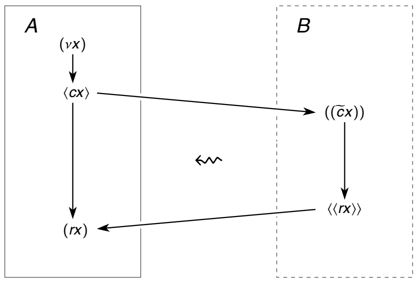
The algebraic fact that

$$\Theta \vdash s \quad \vee \quad \Theta \nvdash s$$

abstracts away partial information leaks.

Task: Refine cryptographic authentication

Challenge-Response pattern



$\text{Local}_A \implies \text{Global}_{AB}$

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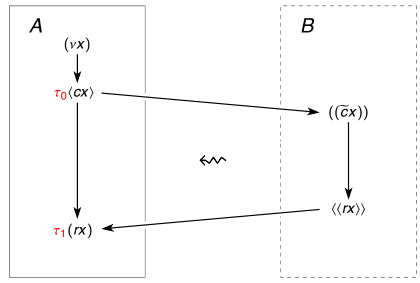
Timed Challenge-Response pattern



$\text{Local}^{\tau}_A \implies \text{Global}^{\tau}_{AB}$

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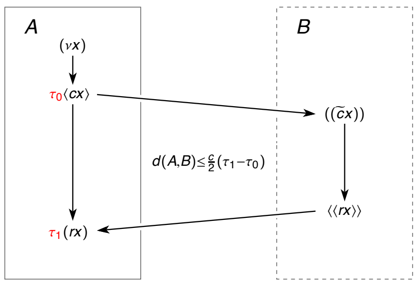

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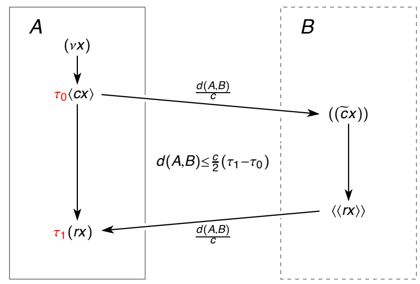

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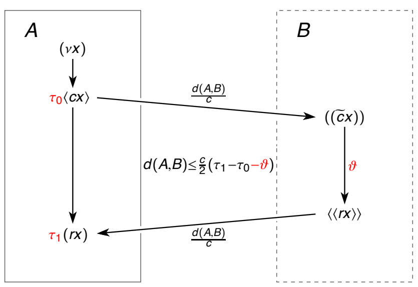

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Problem

- Alice does not know ϑ.

Proximity authentication

Problem

- Alice does not know ϑ.
- Bob looks closer if he is faster.

Proximity authentication

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Problem

- Alice does not know ϑ.
- Bob looks closer if he is faster.

Task

- Minimize ϑ.
- Compute $\widetilde{c}x, k^B \vdash rx$ on-line.

On-line computation

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Definition

A boolean function $f : \mathbb{Z}_2^\ell \longrightarrow \mathbb{Z}_2^\ell$ is

- *on-line* if it returns i-th bit of the output as soon as it has received i-th bit of the input, for $i = 1, 2, \ldots, \ell$

On-line computation

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- *partitioned* if i-th bit of the output only depends on the i-th bit of the input.

On-line computation

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(Blockwise versions are defined analogously.)

Implementing proximity authentication

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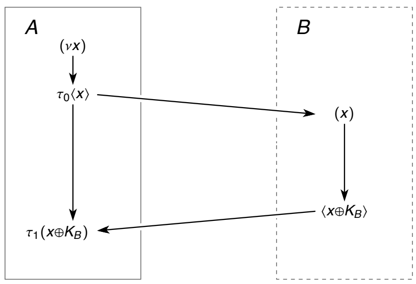
First try: XOR (Brands-Chaum)



Implementing proximity authentication

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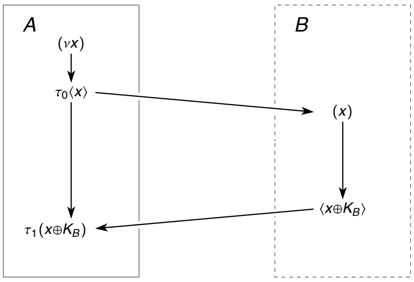
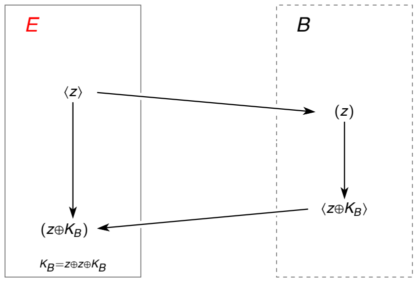


$K_B = H(k^{AB}, y)$ where $(\nu y)_A \to \langle y\rangle_A$

Implementing proximity authentication

Silly attack



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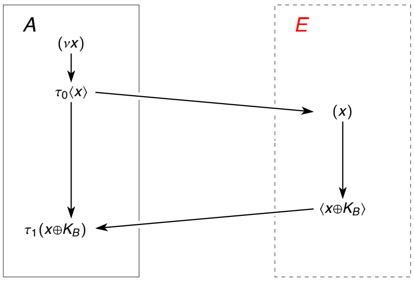

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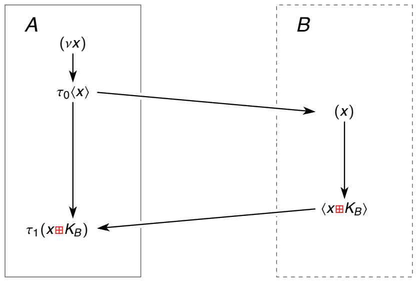
Second try: the Hancke-Kuhn function



$(x \boxplus Y)_i = Y_i^{(x_i)} \qquad Y = Y^{(0)} :: Y^{(1)}$

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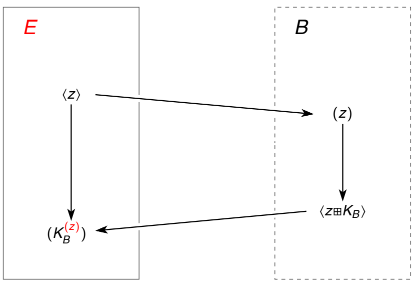
The silly attack only recovers half of the bits



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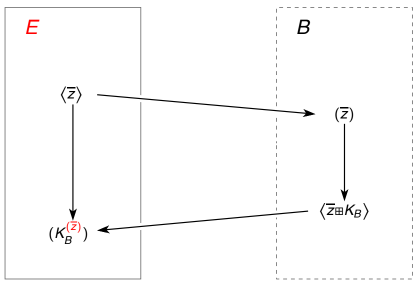
. . . and Eve needs one more challenge. . .



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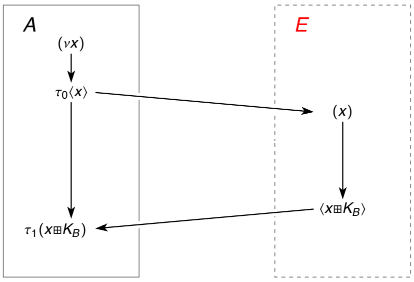
...before she can impersonate Bob



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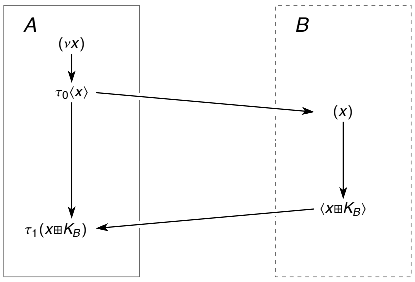
The Hancke-Kuhn protocol: one-time secret



$K_B = H(k^{AB}, y)$ where $(\nu y)_B \to \langle y\rangle_B$

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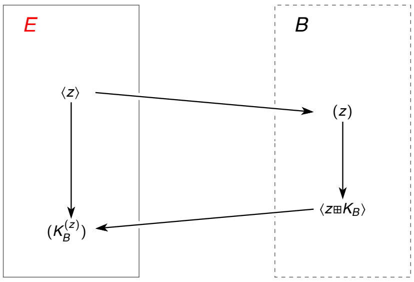
Eve can still get one half of the secret bits



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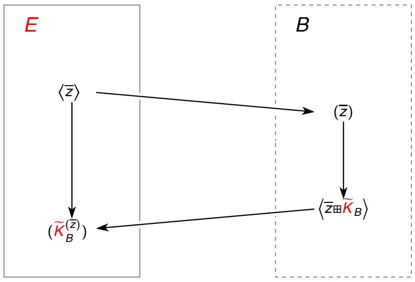
...but not the other half



$\widetilde{K}_B = H(k^{AB}, \widetilde{y})$ where $(\nu \widetilde{y})_B \to \langle \widetilde{y}\rangle_B$

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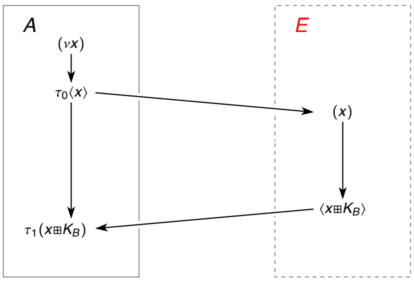
Eve's chance to guess the response



$$\left[z, z \boxplus K_B,\ x \vdash x \boxplus K_B\right] = 2^{-\Delta(z,x)}$$

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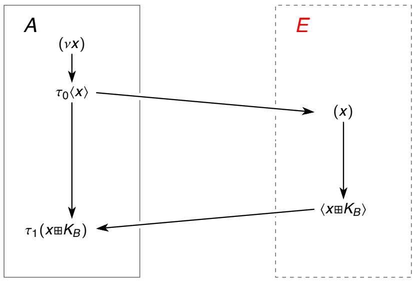
...with the expected value



$$\int_{z\in\mathbb{Z}_2^\ell}\left[z, z \boxplus K_B,\ x \vdash x \boxplus K_B\right] = \left(\frac{3}{4}\right)^\ell$$

Implementing proximity authentication

Facts

- On-line functions always leak information:

$$\left[z, fz, x \vdash fx\right] \quad > \quad \varepsilon(\ell)$$

Implementing proximity authentication

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- On-line functions always leak information:

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- On-line response can be guessed:

$$\{\{k\}, \{z, rz, x\}\}_{z \in Z} \quad \text{guards} \quad rx \quad \text{within} \quad CRP$$

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- Protocols with on-line response do not satisfy Theorem A.

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Proposition

If $f : \mathbb{Z}_2^\ell \longrightarrow \mathbb{Z}_2^\ell$ is bitwise partitioned, then

- $\left[z, f(z), x \vdash f(x)\right] \geq 2^{-\Delta(z,x)}$

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Proposition

If $f : \mathbb{Z}_2^\ell \longrightarrow \mathbb{Z}_2^\ell$ is bitwise partitioned, then

- $\left[z, f(z), x \vdash f(x)\right] \geq 2^{-\Delta(z,x)}$
- $\left[z, f(z), x \vdash f(x)\right] = 2^{-\Delta(z,x)} \iff \forall i. \left[f_i(0) \perp f_i(1)\right]$

Implementing proximity authentication

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D. Pavlovic
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- $\left[z, f(z), x \vdash f(x)\right] = 2^{-\Delta(z,x)} \iff f(x) = x \boxplus \left(f(0^\ell) :: f(1^\ell)\right)$

Implementing proximity authentication

Upshot

- The Hancke-Kuhn protocol is
  - not secure in the symbolic model
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Implementing proximity authentication

Upshot

- The Hancke-Kuhn protocol is
  - not secure in the symbolic model
  - optimal among the partitioned implementations
- Need a better model to evaluate its security.

Bayesian model

- *algebra encoding:* $[\![-]\!] : \mathbb{T}[V] \longrightarrow \wp\{0,1\}^*$

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- *guessing advantage:* $\mathsf{Adv}\left[\Theta \vdash \Gamma\right] = \left[\Theta \vdash \Gamma\right] - \left[\Gamma\right]$
- *independence:* $\left[\Theta \perp \Gamma\right] \iff \mathsf{Adv}\left[\Theta \vdash \Gamma\right] = 0$
  where $\left[\Gamma\right] = \left[\emptyset \vdash \Gamma\right]$

Bayesian model

Lemma

Guessing probability is sub-Bayesian, in the sense

$$\left[\Theta \vdash \Gamma\right] \cdot \left[\Theta, \Gamma \vdash \Xi\right] \;\leq\; \left[\Theta \vdash \Gamma, \Xi\right]$$

which for $\Theta = \emptyset$ and $\left[\Gamma\right] \neq 0$ gives

$$\left[\Gamma \vdash \Xi\right] \;\leq\; \frac{\left[\Gamma, \Xi\right]}{\left[\Gamma\right]}$$

Bayesian model

Remark

Guessing probability is not Bayesian in general:

- $\left[\Gamma\right] \cdot \left[\Gamma \vdash \Theta\right] \neq \left[\Theta\right] \cdot \left[\Theta \vdash \Gamma\right]$
- $\left[\Gamma \perp \Theta\right] \not\Leftrightarrow \left[\Theta \perp \Gamma\right]$

Guessing guards

Definition

For $s \in \mathbb{T}[V]$ and $\mathcal{G}, \mathcal{C} \in \wp\wp\mathbb{T}[V]$ with $s \notin \Theta \in \mathcal{C}$ define

$$\mathcal{G} \;\text{ guards }\; s \;\text{ within }\; \mathcal{C}$$

$$\Updownarrow$$

$$\forall \Theta \in \mathcal{C}. \left(\left[\Theta \vdash s\right] \;\leq\; \sum_{\Gamma \in \mathcal{G}} \left[\Theta \vdash \Gamma\right] \cdot \left[\Theta, \Gamma \vdash s\right] \right.$$

$$\wedge$$

$$\left. \mathsf{Adv}\left[\Theta \vdash s\right] \;\leq\; \bigvee_{\Gamma \in \mathcal{G}} \mathsf{Adv}\left[\Theta \vdash \Gamma\right] \right)$$

Authentication with imperfect cryptography

Theorem B

Suppose that only Bob knows k, such that

- $k, x \vdash rx$
- $\{\{k\}\} \cup \mathcal{X}$ guards rx within CRT

Then

$$\mathrm{Prob}(\mathsf{Global}_{AB} \mid \mathsf{Local}_A) \;\geq\; 1 - \bigvee_{\Theta \in CRT} \int_{\Xi \in \mathcal{X}} \left[\Theta, \Xi \vdash rx\right]$$

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Authentication with imperfect cryptography

Proof

$$\exists\Theta \in \mathcal{C}.\ \Theta \vdash rx, k$$
$$\Downarrow$$
$$\Theta_B \vdash rx$$
$$\Downarrow$$
$$\mathsf{Global}_{AB}$$
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$$\Downarrow$$
$$\exists\Theta \in \mathcal{C}.\ \Theta \vdash rx$$

Authentication with imperfect cryptography

Proof

$$\mathrm{Prob}\left(\exists\Theta \in \mathcal{C}.\ \Theta \vdash rx, k\right)$$
$$\rotatebox{90}{$\geq$}$$
$$\mathrm{Prob}\left(\Theta_B \vdash rx\right) + \varepsilon(\ell)$$
$$\rotatebox{90}{$\geq$}$$
$$\mathrm{Prob}\left(\mathsf{Global}_{AB}\right)$$
$$\rotatebox{90}{$\geq$}$$
$$\mathrm{Prob}\left(\mathsf{Local}_{A}\right)$$
$$\rotatebox{90}{$\geq$}$$
$$\mathrm{Prob}\left(\exists\Theta \in \mathcal{C}.\ \Theta \vdash rx\right)$$

Authentication with imperfect cryptography

Proof

$$\begin{aligned}
\mathrm{Prob}\left(\mathsf{Global}_{AB} \mid \mathsf{Local}_A\right) &= \frac{\mathrm{Prob}\left(\mathsf{Global}_{AB} \wedge \mathsf{Local}_A\right)}{\mathrm{Prob}(\mathsf{Local}_A)} \\
&= \frac{\mathrm{Prob}\left(\mathsf{Global}_{AB}\right)}{\mathrm{Prob}(\mathsf{Local}_A)} \\
&\geq \frac{\mathrm{Prob}\left(\exists\Theta \in \mathcal{C}.\ \Theta \vdash rx, k\right)}{\mathrm{Prob}\left(\exists\Theta \in \mathcal{C}.\ \Theta \vdash rx\right)} - \varepsilon(\ell) \\
&\geq \bigwedge_{\Theta\in\mathcal{C}} \frac{\left[\Theta \vdash rx, k\right]}{\left[\Theta \vdash rx\right]} - \varepsilon(\ell)
\end{aligned}$$

Authentication with imperfect cryptography

Proof

But

$$\bigwedge_{\Theta\in\mathcal{C}} \frac{\left[\Theta \vdash rx, k\right]}{\left[\Theta \vdash rx\right]} \geq 1 - \bigvee_{\Theta\in\mathcal{C}} \int_{\Xi\in\mathcal{X}} \left[\Theta, \Xi \vdash rx\right]$$

follows from

$$\begin{aligned}
\frac{\left[\Theta \vdash rx, k\right]}{\left[\Theta \vdash rx\right]} &\geq \frac{\left[\Theta \vdash k\right]\cdot\left[\Theta, k \vdash rx\right]}{\left[\Theta \vdash k\right]\cdot\left[\Theta, k \vdash rx\right] + \sum_{\Xi\in\mathcal{X}}\left[\Theta \vdash \Xi\right]\cdot\left[\Theta, \Xi \vdash rx\right]} \\
&= 1 - \frac{\sum_{\Xi\in\mathcal{X}}\left[\Theta \vdash \Xi\right]\cdot\left[\Theta, \Xi \vdash rx\right]}{\left[\Theta \vdash rx\right]} \\
&= 1 - \int_{\Xi\in\mathcal{X}} \left[\Theta, \Xi \vdash rx\right]
\end{aligned}$$

Authentication with imperfect cryptography

The case of the Hancke-Kuhn protocol

We have seen that it admits

$$\mathcal{X} = \left\{\{z, z \boxplus K, x\} \mid z \in \mathbb{Z}_2^\ell\right\}$$

as the guessing set, and that

$$\int_{z\in\mathbb{Z}_2^\ell} \left[z, z \boxplus K, x \vdash rx\right] = \left(\frac{3}{4}\right)^\ell$$

is the expected probability of a successful guess.

Authentication with imperfect cryptography

Corollary: Security of the Hancke-Kuhn protocol

Suppose that Alice and Bob share an uncompromised key, and that Bob is honest.

If Alice receives a correct response to her challenge, then the probability that this response originates from Bob is indistinguishable from

$$1 - \left(\frac{3}{4}\right)^\ell$$

where ℓ is the length of the challenge.

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Conclusions and ongoing work

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- This *Bayesian extension of PDL* combines
  - cryptographic formalisms of *provable security* with
  - modules over *assemblies* and *modest sets*.
- Similar combinations simplify reasoning about other cryptographic concepts and frameworks.