Dynamics, robustness and fragility of trust

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Outline

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What is trust?

Alice trusts that Bob will act according to protocol Φ.

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Examples

- shopping: Bob will deliver goods
- marketing: Bob will pay for goods
- access control: Bob will not abuse resources
- key infrastructure: Bob's keys are not compromised

What is trust?

Alice trusts that Bob will act according to protocol Φ.

Examples

- shopping: Bob will deliver goods
- marketing: Bob will pay for goods
- access control: Bob will not abuse resources
- key infrastructure: Bob's keys are not compromised
- Prisoners' Dilemma: Bob will not defect
- Centipede game: . . .
- . . . social cooperation is possible

Modeling Trust

Trust relation $A \xrightarrow[r]{\Phi} B$

- A: trustor
- B: trustee
- Φ: entrusted concept (protocol, task, property)
- r: trust rating

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Views of Trust

Local: trust logics

$A \xrightarrow{\Phi} B$ means that

- A requires Φ
- B guarantees Φ

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Views of Trust

Global: trust networks

$A \xrightarrow[r]{d} B \xrightarrow[s]{d} C \xrightarrow[t]{d} D \xrightarrow[u]{b} K$ means that

- A has a delegation certificate for B
- B has a delegation certificate for C
- C has a delegation certificate for D
- D has a binding certificate for the key K

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Global: trust networks

$A \xrightarrow[r]{d} B \xrightarrow[s]{d} C \xrightarrow[t]{d} D \xrightarrow[u]{b} K$ means that

- A has a delegation certificate for B
- B has a delegation certificate for C
- C has a delegation certificate for D
- D has a binding certificate for the key K
- thus A can use the key K
  - even compute its trust rating $rstu$
- although they had no direct contact

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Adverse selection

	TRUSTE-certified	uncertified
honest	94.6%	97.5%
malicious	5.4%	2.5 %

Table: Trustworthyness of TRUSTE [Edelman 2007]

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Adverse selection

	Google	
	sponsored	organic
top	4.44%	2.73%
top 3	5.33%	2.93 %
top 10	5.89%	2.74 %
top 50	5.93%	3.04 %

Table: Malicious search engine placements [Edelman 2007]

Adverse selection

Yahoo!		
	sponsored	organic
top	6.35%	0.00%
top 3	5.72%	0.35 %
top 10	5.14%	1.47 %
top 50	5.40%	1.55 %

Table: Malicious search engine placements [Edelman 2007]

Adverse selection

Ask		
	sponsored	organic
top	7.99%	3.23%
top 3	7.99%	3.24 %
top 10	8.31%	2.94 %
top 50	8.20%	3.12 %

Table: Malicious search engine placements [Edelman 2007]

Adverse selection

"Pillars of the society" phenomenon

- social hubs are more often corrupt
- the rich are more often thieves
- ...

Questions

- Why does adverse selection happen?
- Can it be eliminated? Limited?
- Can we hedge against it?

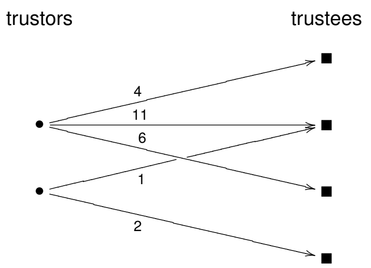
Outline

Trust dynamics

For a moment, we assume that the entrusted property Φ is fixed, and analyze dynamics of trust rating

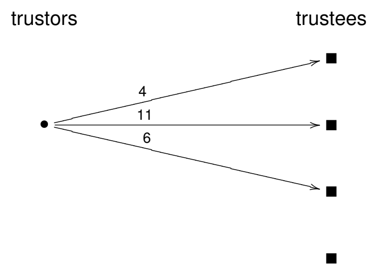
$$A \xrightarrow[r]{} K$$

Trust rating matrix



τ^1	4	11	6	0
τ^2	0	1	0	2

Private trust dynamics



$\tau(t)$	4	11	6	0

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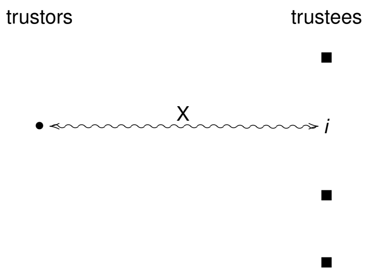
Private trust dynamics



$$\mathrm{Prob}\Big(X(t+1)=i\Big) = C(t)\tau_i(t)$$

(where $C(t) = \frac{1-\alpha}{\sum_{i\in J}\tau_i(t)}$)

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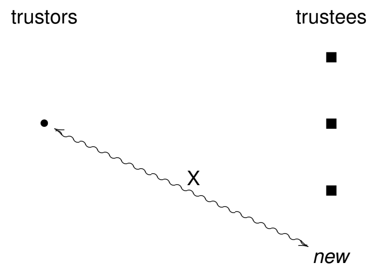
Private trust dynamics



$$\mathrm{Prob}\Big(X(t+1)=\mathit{new}\Big) \quad = \quad \alpha$$

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Private trust dynamics

Trust updating process

$$\tau_i(t+1) \quad = \quad \begin{cases} \tau_i(t) & \text{if } i \neq X(t+1) \\ 0 & \text{if } i = X\text{, not satisfactory} \\ 1 & \text{if } i = X\text{, satisfactory, new} \\ 1+\tau_i(t) & \text{if } i = X\text{, satisfactory, not new} \end{cases}$$

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Task

Estimate

$$w_\ell(t) \quad = \quad \#\{i \in J \mid \tau_i(t) = \ell\}$$

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$$\begin{aligned} w_1(t+1) - w_1(t) &= J \cdot \mathrm{Prob}\big(X(t+1) = i \mid i \text{ new}\big) \cdot \gamma_\perp \\ &\quad - w_1(t) \cdot \mathrm{Prob}\big(X(t+1) = i \mid \tau_i(t) = 1\big) \\ &= J\alpha\gamma_\perp - w_1(t)C(t) \end{aligned}$$

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$$\begin{aligned} w_\ell(t+1) - w_\ell(t) &= w_{\ell-1}(t) \cdot \mathrm{Prob}\big(X(t+1) = i \mid \tau_i(t) = \ell - 1\big) \cdot \gamma_{\ell-1} \\ &\quad - w_\ell(t) \cdot \mathrm{Prob}\big(X(t+1) = i \mid \tau_i(t) = \ell\big) \\ &= w_{\ell-1}(t)C(t)(\ell-1)\gamma_{\ell-1} - w_\ell(t)C(t)\ell \end{aligned}$$

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The system

$$\begin{aligned} \Delta_t w_1(t) &= J\alpha\gamma_\perp - C(t)w_1(t) \\ \Delta_t w_\ell(t) &= w_{\ell-1}(t)C(t)(\ell-1)\gamma_{\ell-1} - w_\ell(t)C(t)\ell \end{aligned}$$

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. . . divided by J becomes

$$\begin{aligned} \Delta_t v_1(t) &= \alpha\gamma_\perp - C(t)v_1(t) \\ \Delta_t v_\ell(t) &= v_{\ell-1}(t)C(t)(\ell-1)\gamma_{\ell-1} - v_\ell(t)C(t)\ell \end{aligned}$$

where $v_\ell(t) = \frac{w_\ell(t)}{J} = \mathrm{Prob}(i \in \mathrm{J} \mid \tau_i(t) = \ell)$
form a stochastic process $v : \mathbb{N} \longrightarrow \mathcal{D}\mathrm{R}$

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. . . and since $v : \mathbb{N} \longrightarrow \mathcal{D}\mathrm{R}$ is a martingale,
it extends to $v : \mathbb{R} \longrightarrow \mathcal{D}\mathrm{R}$ and the system becomes

$$\begin{aligned} \frac{dv_1}{dt} &= \alpha\gamma_\perp - \frac{c}{t}v_1 \\ \frac{dv_\ell}{dt} &= \frac{\gamma_{\ell-1}c(\ell-1)v_{\ell-1} - c\ell v_\ell}{t} \end{aligned}$$

where $C(t) \approx \frac{c}{t}$, for $c = \frac{1-\alpha}{1+\alpha\gamma_\perp}$ (see Appendix)

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The steady state of $v : \mathbb{R} \longrightarrow \mathcal{D}\mathrm{R}$ will be in the form
$v_\ell(t) = t \cdot \upsilon_\ell$, where

$$\begin{aligned} \upsilon_1 &= \alpha\gamma_\perp - c\upsilon_1 \\ \upsilon_\ell &= \gamma_{\ell-1}c(\ell-1)\upsilon_{\ell-1} - c\ell\upsilon_\ell \end{aligned}$$

Trust distribution

The steady state of $v : \mathbb{R} \longrightarrow \mathcal{D}\mathrm{R}$ will be in the form $v_\ell(t) = t \cdot \upsilon_\ell$, where

$$\begin{aligned} \upsilon_1 &= \frac{\alpha\gamma_\perp}{c+1} \\ \upsilon_\ell &= \frac{(\ell-1)\gamma_{\ell-1} c}{\ell c + 1}\, \upsilon_{\ell-1} \end{aligned}$$

Trust distribution

...which expands into

$$\begin{aligned} \upsilon_2 &= \frac{\alpha\gamma_\perp}{c+1} \cdot \frac{\gamma_1 c}{2c+1} \\ \upsilon_3 &= \frac{\alpha\gamma_\perp}{c+1} \cdot \frac{\gamma_1 c}{2c+1} \cdot \frac{2\gamma_2 c}{3c+1} \\ &\vdots \\ \upsilon_n &= \alpha\gamma_\perp \left(\prod_{\ell=1}^{n-1} \gamma_\ell\right) c^{n-1} \cdot \frac{(n-1)!}{\prod_{k=1}^{n}(kc+1)} \\ &= \frac{\alpha\gamma_\perp G_{n-1}}{c} \cdot \frac{(n-1)!}{\prod_{k=1}^{n}\left(k+\frac{1}{c}\right)} \\ &= \frac{\alpha\gamma_\perp G_{n-1}}{c} \cdot \frac{\Gamma(n)\Gamma\left(1+\frac{1}{c}\right)}{\Gamma\left(n+1+\frac{1}{c}\right)} \\ &= \frac{\alpha\gamma_\perp G_{n-1}}{c} \cdot B\left(n, 1+\frac{1}{c}\right) \end{aligned}$$

Trust distribution

The solution

$$\begin{aligned} \upsilon_1 &= \frac{\alpha\gamma_\perp}{c+1} \\ \upsilon_n &= \frac{\alpha\gamma_\perp G_{n-1}}{c} B\left(n, 1+\frac{1}{c}\right) \\ &\overset{n\to\infty}{\longrightarrow} \frac{\alpha\gamma_\perp G}{c} \quad n^{-\left(1+\frac{1}{c}\right)} \end{aligned}$$

where

$$G = \prod_{\ell=1}^{\infty} \gamma_\ell > 0 \text{ follows from}$$

$$\frac{1}{e^{s_\ell}} \leq \gamma_\ell \leq 1 \text{ for some}$$

$$\sum_{\ell=1}^{\infty} s_\ell < \infty$$

Trust distribution

Theorem

The described process of trust building leads, in the long run, to the power law distribution of the number of trustees with the trust rating n

$$w_n \approx \frac{\alpha\gamma_\perp G J}{c} \; n^{-\left(1+\frac{1}{c}\right)}$$

Trust distribution

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provided that the incidence of dishonest principals who act honestly long enough to accumulate a high trust rating — is low enough

Trust distribution

Theorem

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$$w_n \approx \frac{\alpha\gamma_\perp G J}{c} \; n^{-\left(1+\frac{1}{c}\right)}$$

provided that the incidence of dishonest principals who act honestly long enough to accumulate a high trust rating — is low enough (so that $\gamma_\ell \overset{\ell\to\infty}{\longrightarrow} 1$ fast enough)

What does this mean?

Some things have a fixed scale

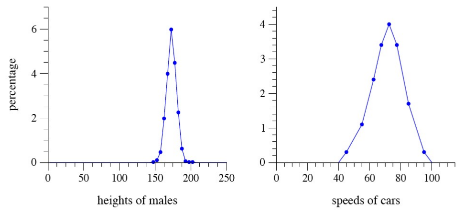

Figure: Normal distribution $f(x) = ae^{-bx^2}$

What does this mean?

Many social phenomena are scale-free

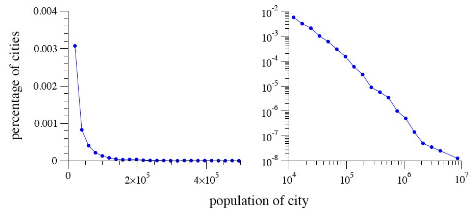

Figure: Power law $w(x) = ax^{-(1+b)}$

Dynamics → robustness → fragility

Dynamics of scale-free distributions

V. Pareto: "The rich get richer"

Dynamics → robustness → fragility

Dynamics of scale-free distributions

V. Pareto: "The rich get richer"

Robustness of scale free distributions

The market is stabilized by the hubs of wealth.

Dynamics → robustness → fragility

Dynamics of scale-free distributions

V. Pareto: "The rich get richer"

Robustness of scale free distributions

The market is stabilized by the hubs of wealth.

Fragility of scale free distributions

Theft is easier when there are very rich people.

Policy guidance

Change dynamics

Modify the process of accumulation to assure a less fragile distribution of trust.

Policy guidance??

Change dynamics

Modify the process of accumulation to assure a less fragile distribution of trust.

Policy guidance??

Change dynamics

Modify the process of accumulation to assure a less fragile distribution of trust.

Moral

Simple social processes lead to complex policy problems.

Private vs public trust

But we only talked about private trust vectors.

Private vs public trust

But we only talked about private trust vectors.

Why is private trust accumulation a social process?

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Private trust process

Public trust process

- Recommender dynamics
- Public trust distribution

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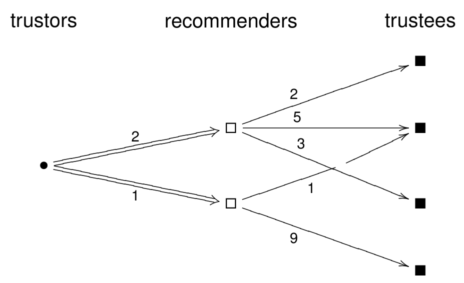
Public trust process

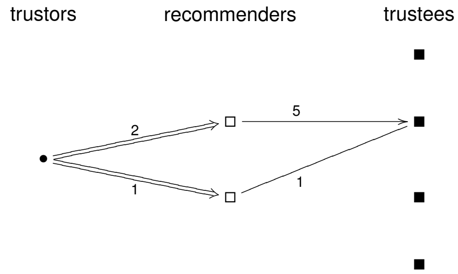
Using recommenders



2	A_1	2	5	3	0
1	A_2	0	1	0	9
σ	τ	4	11	6	9

Public trust process

Using recommenders



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Public trust process

Using recommenders

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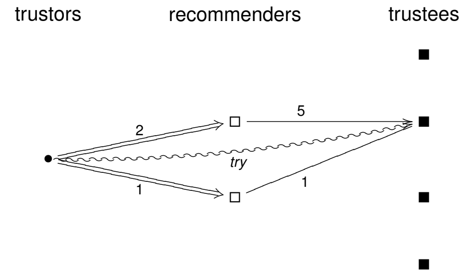

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Public trust process

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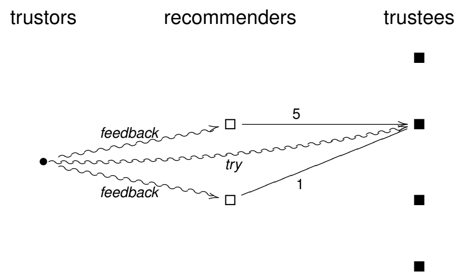

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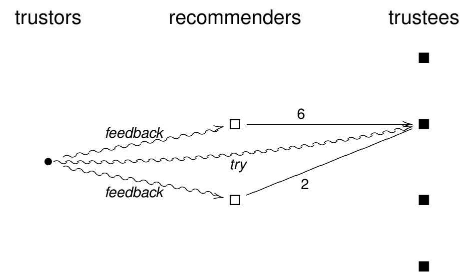

2	A_1	2	6	3	0
1	A_2	0	2	0	9
σ	τ	4	14	6	9

Public trust distribution

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Upshot

Recommenders' public trust vectors also obey the power law distribution.

Recommenders' reputations obey the power law distribution.

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Upshot

Recommenders' public trust vectors also obey the power law distribution.

Recommenders' reputations obey the power law distribution.

Consequence

Adverse selection

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Conclusion

- Trust decisions should not be derived from public trust recommendations alone. They should be based on private trust vectors, that the user should maintain herself.

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Conclusion

- Trust decisions should not be derived from public trust recommendations alone. They should be based on private trust vectors, that the user should maintain herself.
- Public trust recommendations should be used to supplement and refine private trust.

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How?

Idea

- mine for latent trust concepts Φ
- use them to navigate the trust network

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Latent trust concepts

Definition

Let $A = (A_{iu})_{J\times U}$ be a public trust matrix, where U is the set of recommenders and J the set of trustees.

We define the induced *trust concepts* $\Phi_1, \ldots, \Phi_n$ to be the eigenspaces extracted from the singular value decomposition of A. (It is convenient to express them as projectors.)

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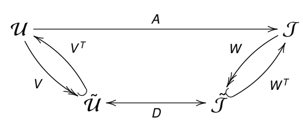
Navigate the trust network

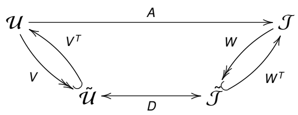
Let the singular value decomposition of A be



where $\mathcal{U} = \mathbb{R}^U$ and $\mathcal{J} = \mathbb{R}^J$.

Navigate the trust network

Let the singular value decomposition of A be



where $\mathcal{U} = \mathbb{R}^U$ and $\mathcal{J} = \mathbb{R}^J$. Then

- each trustor $\tau \in \mathcal{U}$ can derive for
- each principal trust concept Φ_ℓ, and for
- each trustee $j \in J$
- a trust statement $\tau \xrightarrow[r_\ell]{\Phi_\ell} j$ where $r_\ell = \langle \Phi_\ell D V \tau \mid \Phi_\ell W j \rangle$.

Future work

Make this feasible with sparse matrices.