Dynamics of trust
Dusko Pavlovic

Introduction
Private trust
Public trust
Conclusion

Outline

Introduction
Private trust process
Public trust process
Conclusion and future work

What is trust?

Alice trusts that Bob will act according to protocol Φ.

Examples
- shopping: Bob will deliver goods
- marketing: Bob will pay for goods
- access control: Bob will not abuse resources
- key infrastructure: Bob's keys are not compromised

What is trust?

Alice trusts that Bob will act according to protocol Φ.

Examples
- shopping: Bob will deliver goods
- marketing: Bob will pay for goods
- access control: Bob will not abuse resources
- key infrastructure: Bob's keys are not compromised
- Prisoners' Dilemma: Bob will not defect
- Centipede game: 
- ... social cooperation is possible
Modeling Trust

Trust relation $A \xrightarrow{r} B$

- $A$: trustor
- $B$: trustee
- $\Phi$: entrusted concept (protocol, task, property)
- $r$: trust rating

Views of Trust

Local: trust logics
$A \xrightarrow{r} B$ means that
- $A$ requires $\Phi$
- $B$ guarantees $\Phi$

Global: trust networks
$A \xrightarrow{d_r} B \xrightarrow{d_c} C \xrightarrow{d_d} D \xrightarrow{b_u} K$ means that
- $A$ has a delegation certificate for $B$
- $B$ has a delegation certificate for $C$
- $C$ has a delegation certificate for $D$
- $D$ has a binding certificate for the key $K$

Adverse selection

<table>
<thead>
<tr>
<th>TRUSTE-certified</th>
<th>uncertified</th>
</tr>
</thead>
<tbody>
<tr>
<td>honest</td>
<td>94.6%</td>
</tr>
<tr>
<td>malicious</td>
<td>5.4%</td>
</tr>
</tbody>
</table>

Table: Trustworthiness of TRUSTE [Edelman 2007]

Views of Trust

Global: trust networks
$A \xrightarrow{d_r} B \xrightarrow{d_c} C \xrightarrow{d_d} D \xrightarrow{b_u} K$ means that
- $A$ has a delegation certificate for $B$
- $B$ has a delegation certificate for $C$
- $C$ has a delegation certificate for $D$
- $D$ has a binding certificate for the key $K$
- thus $A$ can use the key $K$
  - even compute its trust rating $r_{stu}$
- although they had no direct contact

Adverse selection

<table>
<thead>
<tr>
<th></th>
<th>Google</th>
<th>organic</th>
</tr>
</thead>
<tbody>
<tr>
<td>sponsored</td>
<td>4.44%</td>
<td>2.73%</td>
</tr>
<tr>
<td>top 3</td>
<td>5.33%</td>
<td>2.93%</td>
</tr>
<tr>
<td>top 10</td>
<td>5.89%</td>
<td>2.74%</td>
</tr>
<tr>
<td>top 50</td>
<td>5.93%</td>
<td>3.04%</td>
</tr>
</tbody>
</table>

Table: Malicious search engine placements [Edelman 2007]
Adverse selection

<table>
<thead>
<tr>
<th></th>
<th>sponsored</th>
<th>organic</th>
</tr>
</thead>
<tbody>
<tr>
<td>top</td>
<td>6.35%</td>
<td>0.00%</td>
</tr>
<tr>
<td>top 3</td>
<td>5.72%</td>
<td>0.35%</td>
</tr>
<tr>
<td>top 10</td>
<td>5.14%</td>
<td>1.47%</td>
</tr>
<tr>
<td>top 50</td>
<td>5.40%</td>
<td>1.55%</td>
</tr>
</tbody>
</table>

Table: Malicious search engine placements [Edelman 2007]

Questions

- Why does adverse selection happen?
- Can it be eliminated? Limited?
- Can we hedge against it?

Outline

- Introduction
- Private trust process
  - Trust dynamics
  - Trust distribution
  - Interpretation
- Public trust process
- Conclusion and future work
### Trust rating matrix

<table>
<thead>
<tr>
<th>Trustors</th>
<th>Trustees</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

\[
\tau = \begin{pmatrix} 4 & 11 \\ 6 & 0 \\ 1 & 1 \\ 2 & 3 \end{pmatrix}
\]

### Private trust dynamics

\[
\tau_i(t+1) = \begin{cases} \tau_i(t) & \text{if } i \not\in X(t+1) \\ 0 & \text{if } i = X, \text{ not satisfactory} \\ 1 & \text{if } i = X, \text{ satisfactory, new} \\ 1 + \tau_i(t) & \text{if } i = X, \text{ satisfactory, not new} \end{cases}
\]

### Dynamics of trust

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Introduction

Private trust

Trust updating process

Public trust

Conclusion

### Public trust

### Private trust

### Interpretation

Trust distribution

### Task

Estimate

\[
w_i(t) = \#(j \in J \mid \tau_j(t) = t)
\]
The steady state of $v(t) = t \cdot v_1$ will be in the form

$$v(t) = \frac{\gamma(\ell-1)v_1 - c v_2}{t}$$

where $c = \frac{1}{\ell + \gamma(\ell-1)}$ (see Appendix).
**Trust distribution**

The steady state of \( v : \mathbb{R} \to \mathbb{S} \mathbb{R} \) will be in the form

\[
v(t) = t \cdot v_1,
\]

where

\[
v_1 = \frac{\alpha \gamma_1}{c + 1} \left( (c - 1) \gamma_{c+1} + c \right),
\]

\[
v_2 = \frac{\alpha \gamma_1 \gamma_{c+1}}{c + 1} B \left( n + 1, \frac{1}{c} \right),
\]

where

\[
G = \prod_{k=1}^n \gamma_k > 0 \text{ follows from } \frac{1}{\mathbb{S} \mathbb{R}} \leq \gamma_k \leq 1 \text{ for some } \sum_{k=1}^n \gamma_k < \infty.
\]

**Theorem**

The described process of trust building leads, in the long run, to the power law distribution of the number of trustees with the trust rating \( n \)

\[
w_n = \frac{\alpha \gamma_1 \gamma_{c+1}}{c} n^{-\left(1 + \frac{1}{c}\right)}
\]

provided that the incidence of dishonest principals who act honestly long enough to accumulate a high trust rating — is low enough.

---

**Trust distribution**

\[
\begin{align*}
\tau_1 &= \frac{\alpha \gamma_1}{c + 1} (c - 1) \gamma_{c+1} + c \gamma_{c+1}, \\
\tau_2 &= \frac{\alpha \gamma_1}{c + 1} \frac{\gamma_{c+1}}{2c + 1} (2 \gamma_{c+1} + c), \\
\tau_3 &= \frac{\alpha \gamma_1}{c + 1} \frac{\gamma_{c+1}}{3c + 1} (3 \gamma_{c+1} + c), \\
&\vdots \\
\tau_n &= \frac{\alpha \gamma_1}{c + 1} \frac{\gamma_{c+1}}{(n+1)c + 1} (n \gamma_{c+1} + c).
\end{align*}
\]

... which expands into

\[
\begin{align*}
\tau_1 &= \frac{\alpha \gamma_1}{c + 1} \frac{\gamma_{c+1}}{k+1} (k+1) \gamma_{c+1}, \\
\tau_2 &= \frac{\alpha \gamma_1}{c + 1} \frac{\gamma_{c+1}}{k+1} \frac{(n-1)!}{(n-k-1)!(k+1)!} (k+1), \\
\tau_3 &= \frac{\alpha \gamma_1}{c + 1} \frac{\gamma_{c+1}}{(n+1)! (1 + \frac{1}{c})}, \\
&\vdots \\
\tau_n &= \frac{\alpha \gamma_1}{c + 1} \frac{\gamma_{c+1}}{(n+1)! (1 + \frac{1}{c})}, \\
\end{align*}
\]

**Theorem**

The described process of trust building leads, in the long run, to the power law distribution of the number of trustees with the trust rating \( n \)

\[
w_n = \frac{\alpha \gamma_1 \gamma_{c+1}}{c} n^{-\left(1 + \frac{1}{c}\right)}
\]

provided that the incidence of dishonest principals who act honestly long enough to accumulate a high trust rating — is low enough (so that \( \tau_n \to 1 \) fast enough).
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What does this mean?
Some things have a fixed scale

Figure: Normal distribution $f(x) = ae^{-bx^2}$

What does this mean?
Many social phenomena are scale-free

Figure: Power law $w(x) = ax^{-1+4}$

Dynamics $\rightarrow$ robustness $\rightarrow$ fragility
Dynamics of scale-free distributions
V. Pareto: "The rich get richer"

Robustness of scale-free distributions
The market is stabilized by the hubs of wealth.

Policy guidance
Change dynamics
Modify the process of accumulation to assure a less fragile distribution of trust.

Dynamics $\rightarrow$ robustness $\rightarrow$ fragility
Dynamics of scale-free distributions
V. Pareto: "The rich get richer"

Robustness of scale-free distributions
The market is stabilized by the hubs of wealth.

Fragility of scale-free distributions
Theft is easier when there are very rich people.
Policy guidance??

Change dynamics
Modify the process of accumulation to assure a less fragile distribution of trust.

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Moral
Simple social processes lead to complex policy problems.

Private vs public trust
But we only talked about private trust vectors.

Why is private trust accumulation a social process?

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  - Recommender dynamics
  - Public trust distribution
- Conclusion and future work

Policy guidance??

Change dynamics
Modify the process of accumulation to assure a less fragile distribution of trust.

Public trust process
Using recommenders

trustors
recommenders
trustees

2 1
2 5 3 0
1 A_1 A_2
1 0 9
9
4 11 6
9
Public trust process
Using recommenders

trustors      recommenders      trustees

2     1     5     1

Public trust distribution

Upshot
Recommenders’ public trust vectors also obey the power law distribution.

Consequence
Adverse selection
Introduction

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Beyond

Dynamics of trust

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Private trust

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Conclusion

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Public trust process

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Conclusion

Beyond

Conclusion

I Trust decisions should not be derived from public trust recommendations alone. They should be based on private trust vectors, that the user should maintain herself.

I Public trust recommendations should be used to supplement and refine private trust.

How?

Idea

I mine for latent trust concepts \( \Phi \)

I use them to navigate the trust network

Latent trust concepts

Definition

Let \( A = (A_{ij})_{i \times j} \) be a public trust matrix, where \( U \) is the set of recommenders and \( J \) the set of trustees.

We define the induced trust concepts \( \Phi_1, \ldots, \Phi_n \) to be the eigenspaces extracted from the singular value decomposition of \( A \). (It is convenient to express them as projectors.)

Navigate the trust network

Let the singular value decomposition of \( A \) be

\[
\begin{align*}
U & \rightarrow \Phi \rightarrow V \\
J & \rightarrow W^T \\
\end{align*}
\]

where \( U \in \mathbb{R}^U \) and \( J \in \mathbb{R}^J \).
Navigate the trust network

Let the singular value decomposition of $A$ be

$$
A = U \Sigma V^T
$$

where $U = \mathbb{R}^n$ and $J = \mathbb{R}^l$. Then

- each trustor $\tau \in U$ can derive for
- each principal trust concept $\Phi$, and for
- each trustee $j \in J$
- a trust statement $\tau \xrightarrow{\tau} j$ where $\tau = (\Phi_i D V \tau \mid \Phi_j W_j)$.

Future work

Make this feasible with sparse matrices.