# On the Applicability of Combinatorial Designs to Key Predistribution for Wireless Sensor Networks

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# This presentation

There have been a number of proposals to use combinatorial designs in different ways to construct key predistribution schemes for wireless sensor networks.

We will provide an overview of these techniques and ask:

**Question**

*Are these uses of combinatorial designs appropriate?*

# The Plan

Wireless sensor networks

Combinatorial designs

Key predistribution for WSNs

Direct application of designs

Designs as building blocks

Special networking environments

Conclusions

# Wireless sensor networks

A major computing trend is towards:

- distributed,
- dynamic,
- wireless

networks consisting of lightweight devices, such as wireless sensor networks (WSNs).

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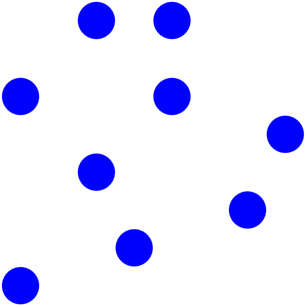

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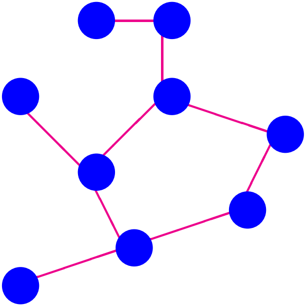

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- Dynamic network structure: Nodes regularly sleep and expire.
- Nodes vulnerable to compromise. Physical security protection such as tamper-resistance is usually not viable, thus nodes can be fairly easily captured.

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2. Communication structure: We will assume that the main aim of any communication in the WSN is to send data from a node to the sink. We will thus not be attempting to set up fully connected subnetworks or establish group keys.
3. No mobility: We will assume that nodes are not mobile after deployment.

# Classification of WSN environments

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3. Fully controlled if the precise location of sensors is known before deployment.

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- There has been some debate about whether public key cryptography can be used on sensor nodes:
  - Even if it can, symmetric cryptography may be preferred for efficiency reasons.
  - Related technology with smaller "sensors" is likely to be proposed.

# Sets systems

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## Definition

*The rank* $k$ *of* $(\mathcal{I}, \mathcal{B})$ *is the size of the largest block in* $\mathcal{B}$ *and we say that* $(\mathcal{I}, \mathcal{B})$ *is uniform if all blocks have size* $k$.

# Designs

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## Definition

*In a dual design, the roles of points and blocks are interchanged.*

# Special designs

## Definition

*Symmetric designs are self-dual and thus have* $v = b$, $k = r$ *and every* $t$ *blocks meeting in* $\lambda$ *points. A symmetric* 2-$(s^2 + s + 1, s^2 + s + 1, s + 1, s + 1, 1)$*-design is known as a projective plane.*

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## Definition

*A transversal design $TD(k, n)$ is set system consisting of $nk$ points such that there exists a partition $\mathcal{H}$ of $\mathcal{I}$ into $k$ groups of size $n$ such that:*

1. *Every $H \in \mathcal{H}$ intersects a block $B \in \mathcal{B}$ in precisely one point;*
2. *Every pair of points from different groups occur together in precisely one block.*

# Key establishment

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- This is especially relevant in applications of symmetric cryptography, where it is necessary to ensure that all parties who are authorised to access (or verify) a cryptographically protected piece of information have the appropriate key.
- Most key establishment mechanisms involve a key management authority (KMA) at some stage in the process.

# Key predistribution

## Definition

*A key predistribution scheme is a key establishment scheme in which the KMA can only be involved in initialisation processes that take place prior to deployment of the network.*

A KMA thus needs to load keys onto nodes prior to deployment using a KPS to determine which keys are allocated to which nodes.

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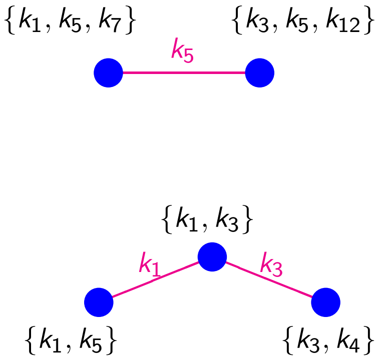
After deployment, two nodes will be able to use a cryptographic service on a network link if they:

1. are in radio communication range of one another; and
2. share at least one key.

# Example

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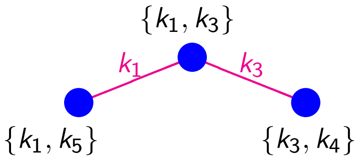

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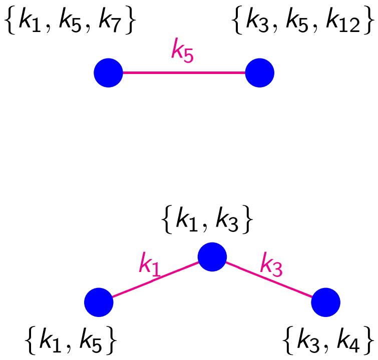


Key predistribution scheme

- nodes are assigned keys before deployment
- nodes that share keys can communicate securely
- two-hop path: nodes communicate via intermediate node

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- The association of $U_j$ with $B_j$ need not be a secret, however the instantiation of $B_j$ by $L_j$ must be.

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- The default method is to broadcast their node allocations to one another, but more efficient techniques can sometimes be found.
- If they have key identifiers in common then a session key can be generated from the common keys associated with these identifiers by means of a suitable key derivation function.

# Phase 3: Path-key establishment



If two nodes fail to identify common keys during shared key discovery then they need to find a secure path between one another that employs intermediate nodes which can.

Obviously, the shorter this secure path the better.

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- Efficiency: Processes such as computation, shared key discovery, and path-key establishment should be as efficient as possible.
- Network size: It is important that a KPS can support as large number of nodes as possible.

# Two trivial KPSs

Single Key KPS: This KPS consists of a single key that is stored by each node in the network. It provides:

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Complete Pairwise Key KPS: In this KPS, a unique key is assigned to each pair of nodes. It provides:

- optimal connectivity
- optimal resilience
- poor storage

# Blom's KPS

- Based on a polynomial $P(x, y) \in \mathrm{GF}(q)[x, y]$ with the property that $P(i, j) = P(j, i)$ for all $i, j \in \mathrm{GF}(q)$.
- Node $U_i$ stores the univariate polynomial $f_i(y) = P(U_i, y)$.
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- optimal connectivity
- if $P$ has degree $w$ then:
  - each node stores $w + 1$ co-efficients
  - full resilience up to capture of $w$ nodes
- very efficient shared key discovery
- does not strictly conform to our model

# Random KPS

- Probabilistic KPS
- Each node draws keys uniformly without replacement from some finite keypool $\mathcal{K}$.
- Properties depend on the number of keys drawn and the size of $\mathcal{K}$.
- Can be further parameterised (for example to require a threshold number of common keys).

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3. Flexibility is attractive:
   - Competing requirements require flexibility
4. Compromise is desirable:
   - Moderation is more likely to be suitable than extremes

# Direct Application of Combinatorial Designs

Combinatorial designs are very natural objects to consider as candidate key rings for KPSs:

- Deterministic
- Rich and well understood structure
- Have long been associated with the building of KPSs

# Prioritising local connectivity

## Theorem

1. *Any block in a* $(v, b, r, k)$*-design* *meets* *at most* $k(r-1)$ *other blocks.*
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- KPSs based on configurations have optimal local connectivity.
- However they offer no guarantees about resilience.

# Prioritising resilience

## Definition

*A* *w-key distribution pattern* *(KDP) is a set system* $(\mathcal{I}, \mathcal{B})$ *with* $|\mathcal{B}| = n$ *such that for any:*

1. *pair* $B_i, B_j \in \mathcal{B}$ *with:* $B_i \cap B_j \neq \emptyset$*; and*
2. $\{B_{l_1}, \ldots, B_{l_w}\} \subseteq \mathcal{B} \setminus \{B_i, B_j\}$*:*

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- KPSs based on KDPs offer optimal resilience if no more than $w$ nodes are compromised.
- A general KDP does not provide any guarantees of connectivity.

# Fully connected configurations

## Theorem

1. *A* $(v, b, r, k)$ *configuration is fully connected precisely when it is the dual of a* $2-(b, v, k, r, 1)$*-design.*
2. *When this happens,* $b \leq k(k-1)+1$.
3. *This bound is met by the projective planes.*

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3. *This bound is met by the projective planes.*

- Optimal connectivity
- Efficient shared key discovery
- Facilitating a very large number of nodes comes at the unattractive cost of relatively large key storage for each node (storage is approximately the square root of the number of nodes).

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- Lack of flexibility:
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- Restrictions on number of nodes:
  - reasonable storage limitations lead to limited number of possible nodes.
- Too much to the extreme:
  - full connectivity is more than we need
  - storage and resilience costs too high

# KPSs from Common Intersection Designs



**Definition**

*Let $(\mathcal{I}, \mathcal{B})$ be a $(v, b, r, k)$-configuration. We say that $(\mathcal{I}, \mathcal{B})$ is a $(v, b, r, k, \mu)$-common intersection design (CID) if for any distinct pair of blocks $B_i, B_j \in \mathcal{B}$ we have:*
$|\{B_k \in \mathcal{B} : B_i \cap B_k \neq \emptyset \text{ and } B_j \cap B_k \neq \emptyset\}| \geq \mu$.

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- CIDs do not have full connectivity
- Optimal CIDs have been constructed from:
  - generalised quadrangles
  - group-divisible designs
  - strongly-regular graphs

# KPSs from transversal designs

- A TD$(k, n)$ is a $(kn, n^2, n, k, k^2 - k)$-CID.
- For any prime $k \leq n$, a useful class of TD$k, n)$s can be constructed known as linear schemes.
- $k$ and $n$ can be varied to produce a range of compromises between the storage $k$, maximum number of nodes $n^2$, local connectivity $k(n + 1)$ and resilience

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- $k$ and $n$ can be varied to produce a range of compromises between the storage $k$, maximum number of nodes $n^2$, local connectivity $k(n+1)$ and resilience
- the local connectivity and resilience can be computed
- very efficient shared-key discovery phase

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However...

- Less known about them
- Still a lack of flexibility
- Still a restriction on the number of nodes

# Designs as Building Blocks



- Combinatorial designs are very natural objects to use as components in the construction of a KPS.
- The resulting KPSs can hopefully be:
  - more flexible
  - inherit the advantages of designs

# Splitting a KPS

1. Start with an original KPS
2. Split nodes by associating each node in the original KPS with a set of nodes in a component KPS.
3. Form a new KPS consisting of:
   - several component KPSs
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- the main gain is an increase in the number of nodes
- the main costs can be either in connectivity or resilience

# Extending a KPS

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As an example:

- KPSs from projective planes were extended by adding random subsets of blocks of the complementary design
- the main gain was more nodes and greater resilience
- the main loss was connectivity (although it was better than the random KPS)

# Packing a KPS

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- the main gain is connectivity
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# Joining KPSs

1. Start with an outer KPS based on set system $(\mathcal{I}, \mathcal{B}^{\text{out}})$
    - Node $U_j$ is associated with the block $B_j^{\text{out}}$
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- KPSs not based on a set system can be used as inner KPSs
  - such as the Blom KPS

# Combinatorial engineering

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Pros:

- some combinatorics can be better than no combinatorics
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  - can potentially gain the "best of all worlds"
- highly flexible
  - KPSs for WSNs are not classical combinatorial objects

# KPSs for Special Networking Environments

Thus far we have discussed uncontrolled and homogeneous KPSs.

- Partially controlled KPSs
- Fully controlled KPSs
- Heterogenous KPSs

# Partially controlled KPSs

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- Keys can be predistributed more efficiently (than for uncontrolled environments):
  - Can deploy a "relaxed" KPS on each group
  - Build in some means of inter-group communication
  - Balanced local connectivity desirable
- Such a KPS can be constructed from joining KPSs, where:
  - outer KPS is based on a resolvable transversal design:
  - two types of inner KPS, both based on Blom KPSs
  - parameters are flexible
  - connectivity and resilience can be traded off against storage
  - efficient shared key discovery inherited from components

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- One option is to only assign shared keys to "neighbours"
  - For dense networks there are more efficient options
- If nodes are deployed in a highly structured physical formation then it is natural to look to combinatorial mathematics for building KPSs:
  - square grids
  - hexagonal grids
  - constructions use distinct difference configurations (which are related to designs)

# Heterogeneous KPSs

In heterogeneous networks not all the nodes have the same capabilities.

Two simple cases are:

1. Simple two-level hierarchies
2. Two-level hierarchies with a backbone

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  - More investigation of **meaningful** construction techniques required
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  - Ad hoc proposals do not necessarily add to the knowledge base

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- Designs are potentially useful building blocks for KPSs:
  - More investigation of **meaningful** construction techniques required
  - Schemes offering good trade-offs are desirable
  - Ad hoc proposals do not necessarily add to the knowledge base
- Some interesting applications of combinatorial designs to special WSN environments.

# Back to the original question

## Question

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*Are combinatorial designs appropriate tools to construct KPSs for WSNs?*

## Answer

- *Yes, in some cases they do provide very interesting constructions.*
- *The most interesting KPSs do not necessarily come directly from designs.*
- *Designs can be used as components for interesting KPSs.*